Computer algebra independent integration tests

4-Trig-functions/4.4-Cotangent/4.4.7-d-trig-^m-a+b-c-cot-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [64]. This is test number [113].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (64)	% 0.00 (0)
Mathematica	% 100.00 (64)	% 0.00 (0)
Maple	% 98.44 (63)	% 1.56 (1)
Maxima	% 32.81 (21)	% 67.19 (43)
Fricas	% 100.00 (64)	% 0.00 (0)
Sympy	% 18.75 (12)	% 81.25 (52)
Giac	% 56.25 (36)	% 43.75 (28)
Mupad	% 60.94 (39)	% 39.06 (25)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

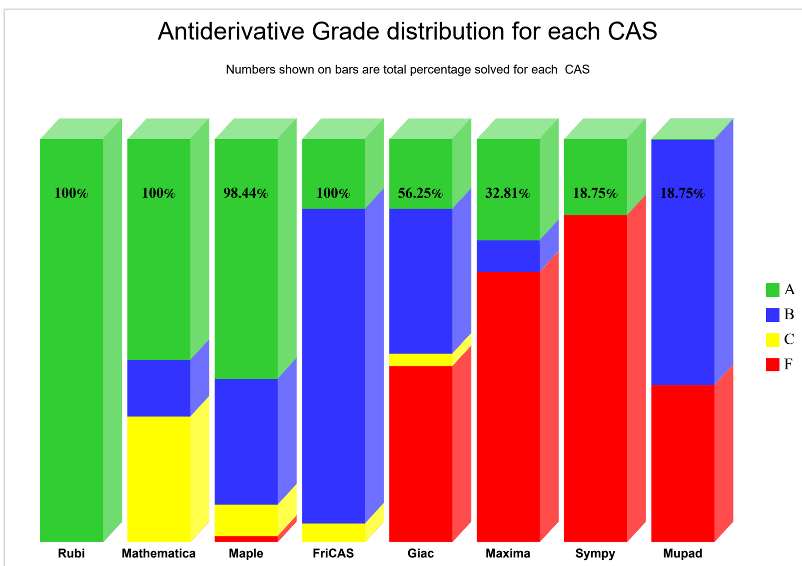
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

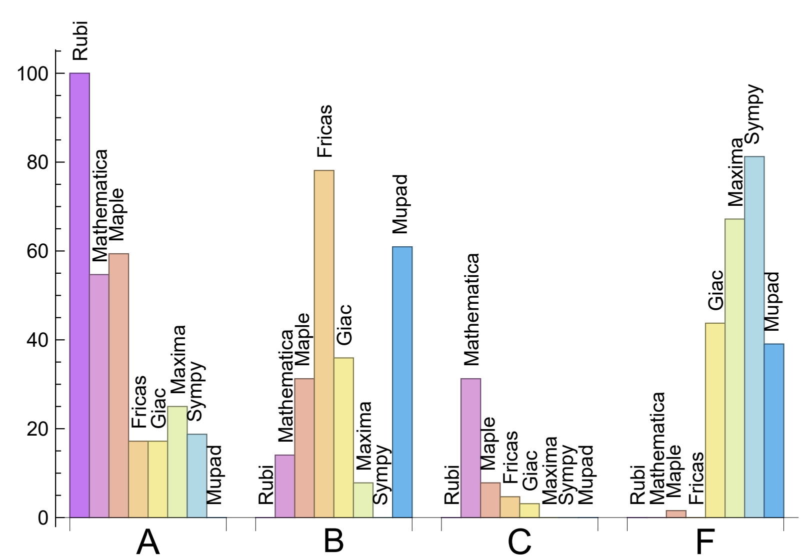
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	54.69	14.06	31.25	0.00
Maple	59.38	31.25	7.81	1.56
Maxima	25.00	7.81	0.00	67.19
Fricas	17.19	78.12	4.69	0.00
Sympy	18.75	0.00	0.00	81.25
Giac	17.19	35.94	3.12	43.75
Mupad	0.00	60.94	0.00	39.06

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Maxima	43	51.16 %	2.33 %	46.51 %
Fricas	0	0.00 %	0.00 %	0.00 %
Sympy	52	100.00 %	0.00 %	0.00 %
Giac	28	28.57 %	7.14 %	64.29 %
Mupad	25	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

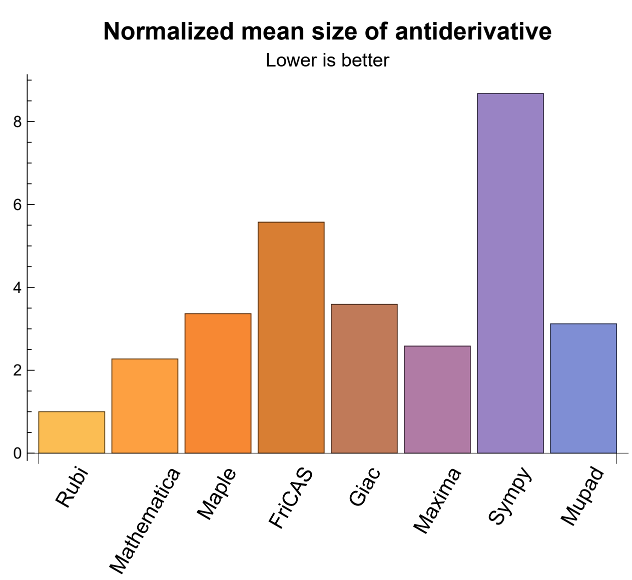
1.3 Performance

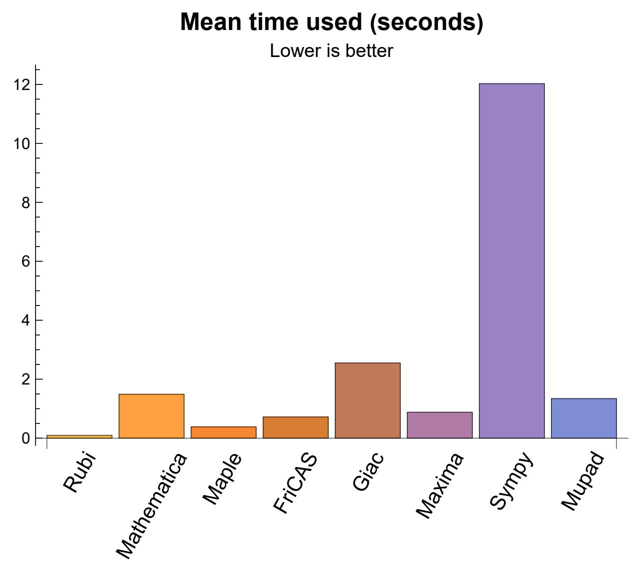
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	70.42	1.00	60.50	1.00
Mathematica	1.49	195.91	2.27	72.00	1.37
Maple	0.38	283.00	3.36	88.00	1.49
Maxima	0.88	86.71	2.58	48.00	1.19
Fricas	0.72	466.34	5.57	337.00	5.38
Sympy	12.03	1053.25	8.68	58.00	1.18
Giac	2.55	340.89	3.59	155.00	2.35
Mupad	1.34	311.87	3.12	47.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {22, 25, 35, 36, 37, 48, 53, 55, 58}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

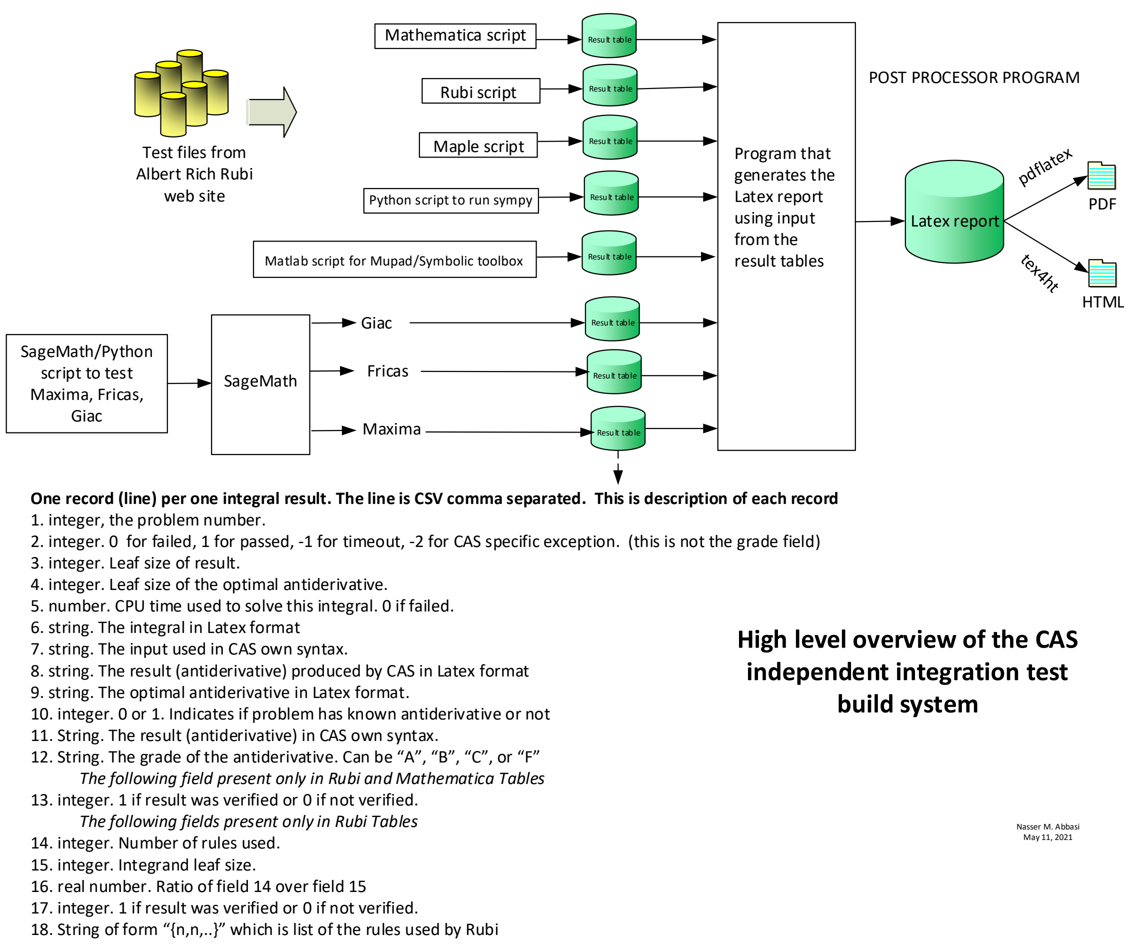
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 3, 4, 5, 6, 7, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 39, 40, 41, 42, 43, 44, 46, 47, 49, 60, 61, 62, 63, 64 }

B grade: { 8, 9, 12, 22, 30, 34, 38, 45, 50 }

C grade: { 2, 23, 24, 25, 31, 32, 33, 35, 36, 37, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59 }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 54, 56, 59, 60, 62 }

B grade: { 6, 7, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 48, 53, 55, 58, 61, 63, 64 }

C grade: { 1, 21, 29, 47, 52 }

F grade: { 57 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 14, 15, 16, 17, 18, 59 }

B grade: { 8, 9, 11, 40, 43 }

C grade: { }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64 }

2.1.5 FriCAS

A grade: { 5, 14, 18, 21, 24, 25, 29, 30, 47, 48, 59 }

B grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 15, 16, 17, 19, 20, 22, 23, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64 }

C grade: { 11, 12, 13 }

F grade: { }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 6, 7, 10, 13, 16, 51, 56, 59 }

B grade: { }

C grade: { }

F grade: { 1, 8, 9, 11, 12, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 57, 58, 60, 61, 62, 63, 64 }

2.1.7 Giac

A grade: { 2, 5, 6, 7, 8, 14, 16, 17, 59, 62, 63 }

B grade: { 3, 4, 20, 21, 23, 24, 25, 34, 35, 36, 37, 38, 41, 43, 44, 46, 47, 50, 55, 58, 60, 61, 64 }

C grade: { 39, 40 }

F grade: { 1, 9, 10, 11, 12, 13, 15, 18, 19, 22, 26, 27, 28, 29, 30, 31, 32, 33, 42, 45, 48, 49, 51, 52, 53, 54, 56, 57 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 26, 28, 29, 34, 38, 39, 40, 42, 43, 44, 46, 47, 49, 51, 52, 54, 56, 57, 59 }

C grade: { }

F grade: { 15, 22, 23, 24, 25, 27, 30, 31, 32, 33, 35, 36, 37, 41, 45, 48, 50, 53, 55, 58, 60, 61, 62, 63, 64 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	148	2494	179	1236	0	0	828
normalized size	1	1.00	0.64	10.70	0.77	5.30	0.00	0.00	3.55
time (sec)	N/A	0.299	0.813	2.507	1.973	1.149	0.000	0.000	0.925

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	31	23	48	22	40	20
normalized size	1	1.00	1.70	1.55	1.15	2.40	1.10	2.00	1.00
time (sec)	N/A	0.013	0.023	0.035	1.144	0.837	0.126	0.192	0.336

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	68	63	127	68	114	45
normalized size	1	1.00	1.51	1.45	1.34	2.70	1.45	2.43	0.96
time (sec)	N/A	0.033	1.171	0.033	0.756	0.817	0.263	0.254	0.118

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	111	116	112	253	126	229	76
normalized size	1	1.00	1.42	1.49	1.44	3.24	1.62	2.94	0.97
time (sec)	N/A	0.047	2.769	0.036	0.928	0.489	0.552	0.371	0.454

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	64	48	252	279	65	41
normalized size	1	1.00	1.00	1.31	0.98	5.14	5.69	1.33	0.84
time (sec)	N/A	0.073	0.057	0.326	0.885	0.461	1.461	0.187	0.124

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	173	115	534	2322	123	119
normalized size	1	1.00	0.93	1.78	1.19	5.51	23.94	1.27	1.23
time (sec)	N/A	0.100	0.907	0.345	0.986	0.511	18.675	0.267	0.786

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	363	228	1068	9629	206	4866
normalized size	1	1.00	0.92	2.42	1.52	7.12	64.19	1.37	32.44
time (sec)	N/A	0.157	0.303	0.352	1.642	0.555	93.929	0.713	3.287

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	19	300	91	0	32	18
normalized size	1	1.00	2.32	0.86	13.64	4.14	0.00	1.45	0.82
time (sec)	N/A	0.018	0.100	0.274	1.101	0.489	0.000	0.183	0.375

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	28	6	35	53	0	0	5
normalized size	1	1.00	5.60	1.20	7.00	10.60	0.00	0.00	1.00
time (sec)	N/A	0.014	0.013	0.229	1.003	0.518	0.000	0.000	0.318

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	10	21	14	0	12
normalized size	1	1.00	1.00	1.08	0.83	1.75	1.17	0.00	1.00
time (sec)	N/A	0.021	0.010	0.179	0.831	0.426	0.342	0.000	0.392

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	32	284	73	0	0	31
normalized size	1	1.00	1.37	0.91	8.11	2.09	0.00	0.00	0.89
time (sec)	N/A	0.026	0.083	0.125	1.008	0.495	0.000	0.000	0.368

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	30	15	17	19	0	0	14
normalized size	1	1.00	2.14	1.07	1.21	1.36	0.00	0.00	1.00
time (sec)	N/A	0.020	0.017	0.128	1.031	0.574	0.000	0.000	0.394

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	14	15	0	13
normalized size	1	1.00	1.00	1.07	0.86	1.00	1.07	0.00	0.93
time (sec)	N/A	0.023	0.004	0.103	0.499	0.841	0.338	0.000	0.682

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	29	24	27	0	30	17
normalized size	1	1.00	0.68	1.04	0.86	0.96	0.00	1.07	0.61
time (sec)	N/A	0.096	0.024	0.296	0.745	0.430	0.000	0.180	0.586

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	38	27	77	0	0	-1
normalized size	1	1.00	1.03	1.23	0.87	2.48	0.00	0.00	-0.03
time (sec)	N/A	0.100	0.033	0.204	0.770	0.749	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	8	27	12	12	10
normalized size	1	1.00	1.00	1.10	0.80	2.70	1.20	1.20	1.00
time (sec)	N/A	0.048	0.011	0.186	0.547	0.522	1.196	0.203	0.476

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	49	56	52	78	0	42	20
normalized size	1	1.00	1.36	1.56	1.44	2.17	0.00	1.17	0.56
time (sec)	N/A	0.087	0.038	0.744	0.653	0.443	0.000	0.192	0.423

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	19	33	18	35	0	0	34
normalized size	1	1.00	0.66	1.14	0.62	1.21	0.00	0.00	1.17
time (sec)	N/A	0.101	0.033	0.573	0.520	0.413	0.000	0.000	0.727

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	84	0	330	0	0	66
normalized size	1	1.00	0.98	1.27	0.00	5.00	0.00	0.00	1.00
time (sec)	N/A	0.116	0.184	0.192	0.000	0.724	0.000	0.000	3.157

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	71	0	248	0	95	53
normalized size	1	1.00	1.00	1.48	0.00	5.17	0.00	1.98	1.10
time (sec)	N/A	0.067	0.027	0.128	0.000	1.511	0.000	0.801	1.170

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	591	0	351	0	187	69
normalized size	1	1.00	1.00	9.85	0.00	5.85	0.00	3.12	1.15
time (sec)	N/A	0.097	0.027	1.074	0.000	0.502	0.000	0.736	0.477

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	2105	174	0	768	0	0	-1
normalized size	1	1.00	23.65	1.96	0.00	8.63	0.00	0.00	-0.01
time (sec)	N/A	0.125	22.398	0.184	0.000	0.494	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	167	137	0	515	0	210	-1
normalized size	1	1.00	2.57	2.11	0.00	7.92	0.00	3.23	-0.02
time (sec)	N/A	0.045	0.408	0.230	0.000	0.505	0.000	2.642	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	752	0	193	0	239	-1
normalized size	1	1.00	0.86	14.75	0.00	3.78	0.00	4.69	-0.02
time (sec)	N/A	0.089	0.097	0.988	0.000	0.577	0.000	0.495	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	174	951	0	239	0	476	-1
normalized size	1	1.00	2.05	11.19	0.00	2.81	0.00	5.60	-0.01
time (sec)	N/A	0.141	1.647	0.770	0.000	0.586	0.000	0.251	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	91	150	0	486	0	0	120
normalized size	1	1.00	1.03	1.70	0.00	5.52	0.00	0.00	1.36
time (sec)	N/A	0.137	0.503	0.132	0.000	0.607	0.000	0.000	11.132

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	253	286	0	1134	0	0	-1
normalized size	1	1.00	1.99	2.25	0.00	8.93	0.00	0.00	-0.01
time (sec)	N/A	0.230	1.194	0.129	0.000	0.681	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	136	0	330	0	0	70
normalized size	1	1.00	0.91	1.97	0.00	4.78	0.00	0.00	1.01
time (sec)	N/A	0.091	0.174	0.093	0.000	0.675	0.000	0.000	3.535

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	2628	0	565	0	0	506
normalized size	1	1.00	1.00	35.04	0.00	7.53	0.00	0.00	6.75
time (sec)	N/A	0.128	0.073	0.759	0.000	1.779	0.000	0.000	0.540

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	222	1276	0	543	0	0	-1
normalized size	1	1.00	2.78	15.95	0.00	6.79	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.741	0.691	0.000	1.433	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	259	462	0	1520	0	0	-1
normalized size	1	1.00	1.51	2.70	0.00	8.89	0.00	0.00	-0.01
time (sec)	N/A	0.186	1.690	0.455	0.000	0.527	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	234	298	0	1071	0	0	-1
normalized size	1	1.00	1.86	2.37	0.00	8.50	0.00	0.00	-0.01
time (sec)	N/A	0.098	1.376	0.379	0.000	0.488	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	202	170	0	703	0	0	-1
normalized size	1	1.00	2.32	1.95	0.00	8.08	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.611	0.420	0.000	0.938	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	111	68	0	239	0	88	41
normalized size	1	1.00	2.36	1.45	0.00	5.09	0.00	1.87	0.87
time (sec)	N/A	0.033	0.413	0.394	0.000	0.479	0.000	4.941	0.854

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	231	104	0	526	0	348	-1
normalized size	1	1.00	2.72	1.22	0.00	6.19	0.00	4.09	-0.01
time (sec)	N/A	0.061	3.635	0.355	0.000	0.624	0.000	10.137	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	367	176	0	898	0	1341	-1
normalized size	1	1.00	2.72	1.30	0.00	6.65	0.00	9.93	-0.01
time (sec)	N/A	0.110	7.938	0.382	0.000	0.678	0.000	13.770	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	2553	284	0	1452	0	3719	-1
normalized size	1	1.00	13.44	1.49	0.00	7.64	0.00	19.57	-0.01
time (sec)	N/A	0.194	14.676	0.381	0.000	0.629	0.000	31.532	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	123	51	0	110	0	257	104
normalized size	1	1.00	2.28	0.94	0.00	2.04	0.00	4.76	1.93
time (sec)	N/A	0.046	0.392	0.230	0.000	1.083	0.000	0.283	0.837

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	62	34	0	68	0	170	88
normalized size	1	1.00	1.94	1.06	0.00	2.12	0.00	5.31	2.75
time (sec)	N/A	0.027	0.070	0.234	0.000	0.408	0.000	0.262	0.963

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	42	31	90	56	0	34	85
normalized size	1	1.00	1.50	1.11	3.21	2.00	0.00	1.21	3.04
time (sec)	N/A	0.019	0.063	0.258	0.506	0.441	0.000	0.210	0.626

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	121	48	0	170	0	179	-1
normalized size	1	1.00	1.98	0.79	0.00	2.79	0.00	2.93	-0.02
time (sec)	N/A	0.043	0.123	0.216	0.000	0.524	0.000	0.665	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	60	35	0	123	0	0	34
normalized size	1	1.00	1.43	0.83	0.00	2.93	0.00	0.00	0.81
time (sec)	N/A	0.025	0.043	0.234	0.000	1.558	0.000	0.000	0.432

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	45	21	143	60	0	45	20
normalized size	1	1.00	1.73	0.81	5.50	2.31	0.00	1.73	0.77
time (sec)	N/A	0.017	0.034	0.247	0.499	0.524	0.000	4.068	0.502

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	44	0	284	0	127	44
normalized size	1	1.00	1.00	0.85	0.00	5.46	0.00	2.44	0.85
time (sec)	N/A	0.098	0.143	0.211	0.000	0.660	0.000	4.910	1.207

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	158	80	0	588	0	0	-1
normalized size	1	1.00	2.47	1.25	0.00	9.19	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.272	0.192	0.000	0.637	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	127	0	70	27
normalized size	1	1.00	1.00	0.88	0.00	3.85	0.00	2.12	0.82
time (sec)	N/A	0.064	0.014	0.151	0.000	0.515	0.000	0.563	0.963

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	376	0	419	0	140	93
normalized size	1	1.00	1.00	6.27	0.00	6.98	0.00	2.33	1.55
time (sec)	N/A	0.094	0.045	0.749	0.000	0.696	0.000	0.385	0.513

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	134	328	0	229	0	0	-1
normalized size	1	1.00	2.48	6.07	0.00	4.24	0.00	0.00	-0.02
time (sec)	N/A	0.091	1.572	0.819	0.000	1.105	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	68	0	385	0	0	52
normalized size	1	1.00	1.00	1.15	0.00	6.53	0.00	0.00	0.88
time (sec)	N/A	0.111	0.213	0.166	0.000	0.453	0.000	0.000	1.917

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	137	99	0	388	0	259	-1
normalized size	1	1.00	2.32	1.68	0.00	6.58	0.00	4.39	-0.02
time (sec)	N/A	0.096	0.732	0.178	0.000	0.744	0.000	1.099	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	56	0	344	48	0	47
normalized size	1	1.00	0.80	1.02	0.00	6.25	0.87	0.00	0.85
time (sec)	N/A	0.075	0.041	0.141	0.000	0.474	10.367	0.000	1.779

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	962	0	863	0	0	1451
normalized size	1	1.00	0.89	11.45	0.00	10.27	0.00	0.00	17.27
time (sec)	N/A	0.131	0.058	0.898	0.000	1.007	0.000	0.000	0.482

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	674	421	0	393	0	0	-1
normalized size	1	1.00	7.33	4.58	0.00	4.27	0.00	0.00	-0.01
time (sec)	N/A	0.154	6.897	0.955	0.000	0.905	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	88	0	698	0	0	88
normalized size	1	1.00	0.84	1.07	0.00	8.51	0.00	0.00	1.07
time (sec)	N/A	0.127	0.099	0.173	0.000	0.680	0.000	0.000	4.236

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	200	166	0	720	0	1025	-1
normalized size	1	1.00	2.13	1.77	0.00	7.66	0.00	10.90	-0.01
time (sec)	N/A	0.124	6.513	0.199	0.000	1.347	0.000	2.398	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	75	0	627	70	0	82
normalized size	1	1.00	0.60	0.96	0.00	8.04	0.90	0.00	1.05
time (sec)	N/A	0.092	0.041	0.138	0.000	0.827	16.770	0.000	4.462

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	78	0	0	1531	0	0	2817
normalized size	1	1.00	0.66	0.00	0.00	12.97	0.00	0.00	23.87
time (sec)	N/A	0.188	0.048	0.604	0.000	0.817	0.000	0.000	1.051

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1450	1040	0	647	0	1242	-1
normalized size	1	1.00	10.28	7.38	0.00	4.59	0.00	8.81	-0.01
time (sec)	N/A	0.241	8.017	1.299	0.000	0.701	0.000	4.941	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	57	37	33	24	34	34	37
normalized size	1	1.00	1.54	1.00	0.89	0.65	0.92	0.92	1.00
time (sec)	N/A	0.062	0.036	0.151	0.417	0.462	0.286	0.425	0.720

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	139	0	1063	0	204	-1
normalized size	1	1.00	0.96	1.54	0.00	11.81	0.00	2.27	-0.01
time (sec)	N/A	0.135	0.154	0.348	0.000	0.830	0.000	0.447	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	167	312	0	1486	0	445	-1
normalized size	1	1.00	1.33	2.48	0.00	11.79	0.00	3.53	-0.01
time (sec)	N/A	0.202	4.396	0.286	0.000	0.851	0.000	1.799	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	0	264	0	58	-1
normalized size	1	1.00	1.00	1.59	0.00	6.44	0.00	1.41	-0.02
time (sec)	N/A	0.069	0.019	0.312	0.000	0.648	0.000	0.383	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	248	0	670	0	111	-1
normalized size	1	1.00	0.99	3.35	0.00	9.05	0.00	1.50	-0.01
time (sec)	N/A	0.111	0.302	0.316	0.000	1.356	0.000	0.444	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	602	0	1365	0	276	-1
normalized size	1	1.00	0.97	5.15	0.00	11.67	0.00	2.36	-0.01
time (sec)	N/A	0.190	0.745	0.333	0.000	0.737	0.000	0.485	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [59] had the largest ratio of [.7500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	12	1.00	25	0.480
2	A	3	2	1.00	12	0.167
3	A	4	3	1.00	14	0.214
4	A	4	3	1.00	14	0.214
5	A	3	3	1.00	14	0.214
6	A	5	5	1.00	14	0.357
7	A	6	6	1.00	14	0.429
8	A	4	4	1.00	10	0.400
9	A	3	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	5	5	1.00	12	0.417
12	A	4	4	1.00	12	0.333
13	A	3	3	1.00	12	0.250
14	A	4	3	1.00	17	0.176
15	A	5	5	1.00	17	0.294
16	A	3	3	1.00	15	0.200
17	A	5	5	1.00	15	0.333
18	A	5	4	1.00	17	0.235
19	A	6	6	1.00	17	0.353
20	A	5	5	1.00	15	0.333
21	A	7	5	1.00	15	0.333
22	A	7	7	1.00	17	0.412
23	A	6	6	1.00	12	0.500
24	A	5	5	1.00	17	0.294
25	A	6	6	1.00	17	0.353
26	A	7	6	1.00	17	0.353
27	A	8	8	1.00	17	0.471
28	A	6	5	1.00	15	0.333
29	A	8	6	1.00	15	0.400
30	A	7	7	1.00	17	0.412
31	A	8	8	1.00	16	0.500
32	A	7	7	1.00	16	0.438
33	A	6	6	1.00	16	0.375
34	A	3	3	1.00	16	0.188
35	A	4	4	1.00	16	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	6	6	1.00	16	0.375
37	A	7	6	1.00	16	0.375
38	A	6	6	1.00	12	0.500
39	A	5	5	1.00	12	0.417
40	A	3	3	1.00	12	0.250
41	A	7	6	1.00	10	0.600
42	A	6	5	1.00	10	0.500
43	A	3	3	1.00	10	0.300
44	A	5	5	1.00	17	0.294
45	A	6	6	1.00	17	0.353
46	A	4	4	1.00	15	0.267
47	A	7	5	1.00	15	0.333
48	A	5	5	1.00	17	0.294
49	A	5	5	1.00	17	0.294
50	A	4	4	1.00	17	0.235
51	A	5	5	1.00	15	0.333
52	A	8	6	1.00	15	0.400
53	A	6	6	1.00	17	0.353
54	A	6	6	1.00	17	0.353
55	A	6	6	1.00	17	0.353
56	A	6	5	1.00	15	0.333
57	A	9	7	1.00	15	0.467
58	A	7	7	1.00	17	0.412
59	A	7	6	1.00	8	0.750
60	A	8	7	1.00	15	0.467
61	A	9	8	1.00	15	0.533
62	A	4	4	1.00	15	0.267
63	A	6	6	1.00	15	0.400
64	A	7	7	1.00	15	0.467

Chapter 3

Listing of integrals

3.1 $\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=233

$$-\frac{(A-C)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}+\frac{(A-C)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\sqrt{b}\,d}-\frac{(A-C)\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{}\right.}{2\sqrt{2}\sqrt{b}\,d}$$

```
[Out] -1/2*(A-C)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)
+1/2*(A-C)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)
-1/4*(A-C)*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^
(1/2)/b^(1/2)+1/4*(A-C)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan
(d*x+c))/d*2^(1/2)/b^(1/2)-2/3*b*C/d/(b*tan(d*x+c))^(3/2)
```

Rubi [A] time = 0.30, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3673, 3629, 12, 16, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(A-C)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}+\frac{(A-C)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\sqrt{b}\,d}-\frac{(A-C)\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{}\right.}{2\sqrt{2}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cot[c + d*x]^2)/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] -(((A - C)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*Sqr
t[b]*d)) + ((A - C)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sq
rt[2]*Sqrt[b]*d) - ((A - C)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sq
rt[b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[b]*d) + ((A - C)*Log[Sqrt[b] + Sqrt[b]
*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[b]*d) - (2*b
*C)/(3*d*(b*Tan[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
```

```
^2 + b^2, 0]
```

Rule 3673

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx = b^2\int \frac{C+A\tan^2(c+dx)}{(b\tan(c+dx))^{5/2}}\,dx$$
$$= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}} + \int \frac{b(A-C)\tan(c+dx)}{(b\tan(c+dx))^{3/2}}\,dx$$
$$= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}} + (b(A-C))\int \frac{\tan(c+dx)}{(b\tan(c+dx))^{3/2}}\,dx$$
$$= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}} + (A-C)\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$
$$= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}} + \frac{(b(A-C))\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}} + \frac{(2b(A-C))\,\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$
$$= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}} + \frac{(A-C)\,\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{(A-C)}{}$$
$$= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}} + \frac{(A-C)\,\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}\,x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d} + -$$
$$= -\frac{(A-C)\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}\,d} + \frac{(A-C)\log\left(\sqrt{b}+\right.}{}$$
$$= -\frac{(A-C)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d} + \frac{(A-C)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d} - \frac{(A-}{}$$

Mathematica [A] time = 0.81, size = 148, normalized size = 0.64

$$\frac{-3\sqrt{2}(A-C)\sqrt{\tan(c+dx)}\left(2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)+\log\left(\tan(c+\right.\right.}{12d\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cot[c + d*x]^2)/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] (-8*C*Cot[c + d*x] - 3*Sqrt[2]*(A - C)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]])*Sqrt[Tan[c + d*x]])/(12*d*Sqrt[b*Tan[c + d*x]])
```

fricas [B] time = 1.15, size = 1236, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*C*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c)^2 + 12*(sqrt(2)*b*
d*cos(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4
)/(b^2*d^4))^(1/4)*arctan((sqrt(2)*(A - C)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*
x + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) + sqr
t(2)*b*d^3*sqrt((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(
b^2*d^4))*cos(d*x + c) + sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos(d*x +
c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*cos(d*x +
 c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c))*((A^4 - 4*A^3*C + 6
*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) + A^4 - 4*A^3*C + 6*A^2*C^2 - 4*
A*C^3 + C^4)/(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)) + 12*(sqrt(2)*b*d
*cos(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)
/(b^2*d^4))^(1/4)*arctan((sqrt(2)*(A - C)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*x
 + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) + sqrt
(2)*b*d^3*sqrt((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b
^2*d^4))*cos(d*x + c) - sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c
))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*cos(d*x +
c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c))*((A^4 - 4*A^3*C + 6*
A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) - A^4 + 4*A^3*C - 6*A^2*C^2 + 4*A
*C^3 - C^4)/(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)) + 3*(sqrt(2)*b*d*c
os(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(
b^2*d^4))^(1/4)*log((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^
4)/(b^2*d^4))*cos(d*x + c) + sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos(d*
x + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*cos(d
*x + c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c)) - 3*(sqrt(2)*b*
d*cos(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4
)/(b^2*d^4))^(1/4)*log((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 +
 C^4)/(b^2*d^4))*cos(d*x + c) - sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos
(d*x + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*co
s(d*x + c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c)))/(b*d*cos(d*
x + c)^2 - b*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C \cot(dx+c)^2 + A}{\sqrt{b \tan(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cot(d*x + c)^2 + A)/sqrt(b*tan(d*x + c)), x)
```

maple [C] time = 2.51, size = 2494, normalized size = 10.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*C*cos(d*x+c)*EllipticPi((-(-
sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(
-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*A*cos(d*x+c)*Ell
ipticPi((-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*A*El
```

```
lipticPi((-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*C*c
os(d*x+c)*EllipticPi((-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*
x+c)-3*A*cos(d*x+c)*EllipticPi((-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*sin(d*x+c)-3*A*cos(d*x+c)*EllipticPi((-(-sin(d*x+c)-1+cos(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*sin(d*x+c)+6*A*cos(d*x+c)*EllipticF((-(-sin(d*x+c)-1+cos(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*sin(d*x+c)+3*I*A*cos(d*x+c)*EllipticPi((-(-sin(d*x+c)-1+cos(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*I*C*EllipticPi((-(-sin(d*x+c)-1+cos(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*C*cos(d*x+c)*EllipticPi((-(-sin(d*x+
c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x
+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*C*cos(d*x+c)*EllipticPi((-
(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-6*C*cos(d*x+c)*Ell
ipticF((-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*I*A*EllipticPi((-
(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*I*C*EllipticPi((
-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*A*EllipticPi((-
(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-3*A*EllipticPi((-(
-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+6*A*EllipticF((-(-s
in(d*x+c)-1+cos(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*C*EllipticPi((-(-sin(d*x+c)-1
+cos(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+3*C*EllipticPi((-(-sin(d*x+c)-1+
cos(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-6*C*EllipticF((-(-sin(d*x+c)-1+co
s(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-sin(d*x+c)-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*sin(d*x+c)+2*C*2^(1/2)*cos(d*x+c)^2)/sin(d*x+c)^5/cos(d*
x+c)/(b*sin(d*x+c)/cos(d*x+c))^(1/2)*2^(1/2)
```

maxima [A] time = 1.97, size = 179, normalized size = 0.77

$$\frac{3\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}\sqrt{b}\log\left(b\tan\left(d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*t
an(d*x + c)))/sqrt(b)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*sqrt(b)*log(b*tan(d*x + c
) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*sqrt(b)*log(b*tan(d
*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b))*(A - C) - 8*C*b^2/(b*t
an(d*x + c))^(3/2))/(b*d)
```

mupad [B] time = 0.92, size = 828, normalized size = 3.55

$$-\frac{2\,C\,b}{3\,d\,{\left(b\tan\left(c+d\,x\right)\right)}^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left(\frac{\frac{{\left(-1\right)}^{1/4}\,\left(A-C\right)\left(\sqrt{b\tan(c+d\,x)}\,\left(16\,A^2\,b^2\,d^3-32\,A\,C\,b^2\,d^3+16\,C^2\,b^2\,d^3\right)-\frac{{\left(-1\right)}^{1/4}\,\left(A-C\right)\left(32\,A\,b^3\,d^4-32\,C\,b^3\,d^4\right)}{2\,\sqrt{b}\,d}\right)1\mathrm{i}}{2\,\sqrt{b}\,d}+\cdots}{\frac{{\left(-1\right)}^{1/4}\,\left(A-C\right)\left(\sqrt{b\tan(c+d\,x)}\,\left(16\,A^2\,b^2\,d^3-32\,A\,C\,b^2\,d^3+16\,C^2\,b^2\,d^3\right)-\frac{{\left(-1\right)}^{1/4}\,\left(A-C\right)\left(32\,A\,b^3\,d^4-32\,C\,b^3\,d^4\right)}{2\,\sqrt{b}\,d}\right)}{2\,\sqrt{b}\,d}-\cdots}\right.}{\sqrt{b}\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cot(c + d*x)^2)/(b*tan(c + d*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*atan((((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d
^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 -
 32*C*b^3*d^4))/(2*b^(1/2)*d))*1i)/(2*b^(1/2)*d) + ((-1)^(1/4)*(A - C)*((b*
tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((
-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4))/(2*b^(1/2)*d))*1i)/(2*b^(1
/2)*d))/(((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C
^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3
*d^4))/(2*b^(1/2)*d)))/(2*b^(1/2)*d) - ((-1)^(1/4)*(A - C)*((b*tan(c + d*x)
)^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-1)^(1/4)*(A
 - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4))/(2*b^(1/2)*d)))/(2*b^(1/2)*d)))*(A - C
)*1i)/(b^(1/2)*d) - (2*C*b)/(3*d*(b*tan(c + d*x))^(3/2)) + ((-1)^(1/4)*atan
((((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*
d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4)*1
i)/(2*b^(1/2)*d)))/(2*b^(1/2)*d) + ((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1
/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-1)^(1/4)*(A - C
)*(32*A*b^3*d^4 - 32*C*b^3*d^4)*1i)/(2*b^(1/2)*d)))/(2*b^(1/2)*d))/(((-1)^(
1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*
A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4)*1i)/(2*b^(
1/2)*d))*1i)/(2*b^(1/2)*d) - ((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(1
6*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-1)^(1/4)*(A - C)*(32*
A*b^3*d^4 - 32*C*b^3*d^4)*1i)/(2*b^(1/2)*d))*1i)/(2*b^(1/2)*d)))*(A - C))/(
b^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+C\cot^2\left(c+dx\right)}{\sqrt{b\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)**2)/(b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cot(c + d*x)**2)/sqrt(b*tan(c + d*x)), x)
```

3.2 $\int \left(a+b\cot^2(c+dx)\right) dx$

Optimal. Leaf size=20

$$ax-\frac{b\cot(c+dx)}{d}-bx$$

```
[Out] a*x-b*x-b*cot(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax-\frac{b\cot(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cot[c + d*x]^2,x]
```

```
[Out] a*x - b*x - (b*Cot[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cot^2(c+dx)\right) dx &= ax+b\int \cot^2(c+dx)\,dx\\ &= ax-\frac{b\cot(c+dx)}{d}-b\int 1\,dx\\ &= ax-bx-\frac{b\cot(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.02, size = 34, normalized size = 1.70

$$ax-\frac{b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cot[c + d*x]^2,x]
```

```
[Out] a*x - (b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d
```

fricas [B] time = 0.84, size = 48, normalized size = 2.40

$$\frac{(a-b)dx\sin(2\,dx+2\,c)-b\cos(2\,dx+2\,c)-b}{d\sin(2\,dx+2\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((a - b)*d*x*sin(2*d*x + 2*c) - b*cos(2*d*x + 2*c) - b)/(d*sin(2*d*x + 2*c)
)
```

giac [A] time = 0.19, size = 40, normalized size = 2.00

$$ax - \frac{\left(2\,dx + 2\,c + \frac{1}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)} - \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)^2,x, algorithm="giac")
[Out] a*x - 1/2*(2*d*x + 2*c + 1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))*b/d
```

maple [A] time = 0.04, size = 31, normalized size = 1.55

$$ax + \frac{b\left(-\cot\left(dx + c\right) + \frac{\pi}{2} - \operatorname{arccot}\left(\cot\left(dx + c\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cot(d*x+c)^2,x)
[Out] a*x+b/d*(-cot(d*x+c)+1/2*Pi-arccot(cot(d*x+c)))
```

maxima [A] time = 1.14, size = 23, normalized size = 1.15

$$ax - \frac{\left(dx + c + \frac{1}{\tan(dx+c)}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)^2,x, algorithm="maxima")
[Out] a*x - (d*x + c + 1/tan(d*x + c))*b/d
```

mupad [B] time = 0.34, size = 20, normalized size = 1.00

$$x\,\left(a - b\right) - \frac{b\cot\left(c + d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cot(c + d*x)^2,x)
[Out] x*(a - b) - (b*cot(c + d*x))/d
```

sympy [A] time = 0.13, size = 22, normalized size = 1.10

$$ax + b\left(\begin{cases} -x - \frac{\cot(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\cot^2(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)**2,x)
[Out] a*x + b*Piecewise((-x - cot(c + d*x)/d, Ne(d, 0)), (x*cot(c)**2, True))
```

3.3 $\int \left(a+b\cot^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$-\frac{b(2a-b)\cot(c+dx)}{d}+x(a-b)^2-\frac{b^2\cot^3(c+dx)}{3d}$$

```
[Out] (a-b)^2*x-(2*a-b)*b*cot(d*x+c)/d-1/3*b^2*cot(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = $\{3661, 390, 203\}$

$$-\frac{b(2a-b)\cot(c+dx)}{d}+x(a-b)^2-\frac{b^2\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^2,x]
```

```
[Out] (a - b)^2*x - ((2*a - b)*b*Cot[c + d*x])/d - (b^2*Cot[c + d*x]^3)/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\cot^2(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2}\,dx,x,\cot(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)dx,x,\cot(c+dx)\right)}{d}\\
&= -\frac{(2a-b)b\cot(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}-\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\cot(c+dx)\right)}{d}\\
&= (a-b)^2x-\frac{(2a-b)b\cot(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 1.17, size = 71, normalized size = 1.51

$$-\frac{\cot(c+dx)\left(b\left(6a+b\cot^2(c+dx)-3b\right)+3(a-b)^2\sqrt{-\tan^2(c+dx)}\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^2,x]
```

```
[Out] -1/3*(Cot[c + d*x]*(b*(6*a - 3*b + b*Cot[c + d*x]^2) + 3*(a - b)^2*ArcTanh[
Sqrt[-Tan[c + d*x]^2]]*Sqrt[-Tan[c + d*x]^2]))/d
```

fricas [B] time = 0.82, size = 127, normalized size = 2.70

$$\frac{2\,b^2\cos\left(2\,dx+2\,c\right)-2\left(3\,ab-2\,b^2\right)\cos\left(2\,dx+2\,c\right)^2+6\,ab-2\,b^2+3\left(\left(a^2-2\,ab+b^2\right)dx\cos\left(2\,dx+2\,c\right)-\left(a^2-\right.\right.}{3\left(d\cos\left(2\,dx+2\,c\right)-d\right)\sin\left(2\,dx+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*cos(2*d*x + 2*c) - 2*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 6*a*b
- 2*b^2 + 3*((a^2 - 2*a*b + b^2)*d*x*cos(2*d*x + 2*c) - (a^2 - 2*a*b + b^2)
*d*x)*sin(2*d*x + 2*c))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))
```

giac [B] time = 0.25, size = 114, normalized size = 2.43

$$\frac{b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\left(a^2-2\,ab+b^2\right)(dx+c)-\frac{24\,ab\tan\left(\frac{1}{2}\,dx+\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(b^2*tan(1/2*d*x + 1/2*c)^3 + 24*a*b*tan(1/2*d*x + 1/2*c) - 15*b^2*tan
(1/2*d*x + 1/2*c) + 24*(a^2 - 2*a*b + b^2)*(d*x + c) - (24*a*b*tan(1/2*d*x
+ 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + b^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.03, size = 68, normalized size = 1.45

$$\frac{-\frac{\left(\cot^3(dx+c)\right)b^2}{3}-2ab\cot\left(dx+c\right)+b^2\cot\left(dx+c\right)+\left(-a^2+2ab-b^2\right)\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(dx+c\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/3*cot(d*x+c)^3*b^2-2*a*b*cot(d*x+c)+b^2*cot(d*x+c)+(-a^2+2*a*b-b^2)
*(1/2*Pi-arccot(cot(d*x+c))))
```

maxima [A] time = 0.76, size = 63, normalized size = 1.34

$$a^2x-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab}{d}+\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(d*x + c + 1/tan(d*x + c))*a*b/d + 1/3*(3*d*x + 3*c + (3*tan(d*x
+ c)^2 - 1)/tan(d*x + c)^3)*b^2/d
```

mupad [B] time = 0.12, size = 45, normalized size = 0.96

$$x\,(a-b)^2-\frac{b^2\cot\left(c+d\,x\right)^3}{3\,d}-\frac{b\cot\left(c+d\,x\right)\,\left(2\,a-b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^2,x)
```

```
[Out] x*(a - b)^2 - (b^2*cot(c + d*x)^3)/(3*d) - (b*cot(c + d*x)*(2*a - b))/d
```

sympy [A] time = 0.26, size = 68, normalized size = 1.45

$$\begin{cases} a^2x-2abx-\frac{2ab\cot\left(c+dx\right)}{d}+b^2x-\frac{b^2\cot^3\left(c+dx\right)}{3d}+\frac{b^2\cot\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x\left(a+b\cot^2\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x - 2*a*b*cot(c + d*x)/d + b**2*x - b**2*cot(c +
d*x)**3/(3*d) + b**2*cot(c + d*x)/d, Ne(d, 0)), (x*(a + b*cot(c)**2)**2, Tr
ue))
```

3.4 $\int \left(a+b\cot^2(c+dx)\right)^3 dx$

Optimal. Leaf size=78

$$-\frac{b\left(3a^2-3ab+b^2\right)\cot(c+dx)}{d}-\frac{b^2(3a-b)\cot^3(c+dx)}{3d}+x(a-b)^3-\frac{b^3\cot^5(c+dx)}{5d}$$

```
[Out] (a-b)^3*x-b*(3*a^2-3*a*b+b^2)*cot(d*x+c)/d-1/3*(3*a-b)*b^2*cot(d*x+c)^3/d-1
/5*b^3*cot(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$-\frac{b\left(3a^2-3ab+b^2\right)\cot(c+dx)}{d}-\frac{b^2(3a-b)\cot^3(c+dx)}{3d}+x(a-b)^3-\frac{b^3\cot^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^3,x]
```

```
[Out] (a - b)^3*x - (b*(3*a^2 - 3*a*b + b^2)*Cot[c + d*x])/d - ((3*a - b)*b^2*Cot
[c + d*x]^3)/(3*d) - (b^3*Cot[c + d*x]^5)/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\cot^2(c+dx)\right)^3 dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(b\left(3a^2-3ab+b^2\right)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right)dx, x, \cot(c+dx)\right)}{d}\\
&= -\frac{b\left(3a^2-3ab+b^2\right)\cot(c+dx)}{d}-\frac{(3a-b)b^2\cot^3(c+dx)}{3d}-\frac{b^3\cot^5(c+dx)}{5d}-\frac{(a-\cdots}{\cdots}\\
&= (a-b)^3x-\frac{b\left(3a^2-3ab+b^2\right)\cot(c+dx)}{d}-\frac{(3a-b)b^2\cot^3(c+dx)}{3d}-\frac{b^3\cot^5(c+d\cdots}{5d}
\end{aligned}$$

Mathematica [A] time = 2.77, size = 111, normalized size = 1.42

$$-\frac{\cot^5(c+dx)\left(b\left(15\left(3a^2-3ab+b^2\right)\tan^4(c+dx)+5b(3a-b)\tan^2(c+dx)+3b^2\right)+\frac{15(a-b)^3\tan^8(c+dx)\tanh^{-1}\left(\sqrt{\ldots}\right.}{\left(-\tan^2(c+dx)\right)^{3/2}}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^3,x]
```

```
[Out] -1/15*(Cot[c + d*x]^5*((15*(a - b)^3*ArcTanh[Sqrt[-Tan[c + d*x]^2]]*Tan[c +
 d*x]^8)/(-Tan[c + d*x]^2)^(3/2) + b*(3*b^2 + 5*(3*a - b)*b*Tan[c + d*x]^2
+ 15*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x]^4)))/d
```

fricas [B] time = 0.49, size = 253, normalized size = 3.24

$$-\frac{\left(45\,a^2b-60\,ab^2+23\,b^3\right)\cos\left(2\,dx+2\,c\right)^3+45\,a^2b-30\,ab^2+13\,b^3-\left(45\,a^2b-30\,ab^2+b^3\right)\cos\left(2\,dx+2\,c\right)^2\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((45*a^2*b - 60*a*b^2 + 23*b^3)*cos(2*d*x + 2*c)^3 + 45*a^2*b - 30*a*
b^2 + 13*b^3 - (45*a^2*b - 30*a*b^2 + b^3)*cos(2*d*x + 2*c)^2 - (45*a^2*b -
 60*a*b^2 + 11*b^3)*cos(2*d*x + 2*c) - 15*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
d*x*cos(2*d*x + 2*c)^2 - 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cos(2*d*x +
2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sin(2*d*x + 2*c))/((d*cos(2*d*x
 + 2*c)^2 - 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c))
```

giac [B] time = 0.37, size = 229, normalized size = 2.94

$$\frac{3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+60\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-900\,ab^2\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^3*tan(1/2*d*x + 1/2*c)^5 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 35*
b^3*tan(1/2*d*x + 1/2*c)^3 + 720*a^2*b*tan(1/2*d*x + 1/2*c) - 900*a*b^2*tan
(1/2*d*x + 1/2*c) + 330*b^3*tan(1/2*d*x + 1/2*c) + 480*(a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*(d*x + c) - (720*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 900*a*b^2*tan(1
/2*d*x + 1/2*c)^4 + 330*b^3*tan(1/2*d*x + 1/2*c)^4 + 60*a*b^2*tan(1/2*d*x +
 1/2*c)^2 - 35*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*b^3)/tan(1/2*d*x + 1/2*c)^5)/
d
```

maple [A] time = 0.04, size = 116, normalized size = 1.49

$$\frac{-\frac{b^3\left(\cot^5(dx+c)\right)}{5}-\left(\cot^3\left(dx+c\right)\right)a\,b^2+\frac{b^3\left(\cot^3(dx+c)\right)}{3}-3a^2b\cot\left(dx+c\right)+3b^2a\cot\left(dx+c\right)-b^3\cot\left(dx+c\right)+\left(-\ldots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-1/5*b^3*cot(d*x+c)^5-cot(d*x+c)^3*a*b^2+1/3*b^3*cot(d*x+c)^3-3*a^2*b*
cot(d*x+c)+3*b^2*a*cot(d*x+c)-b^3*cot(d*x+c)+(-a^3+3*a^2*b-3*a*b^2+b^3)*(1/
2*Pi-arccot(cot(d*x+c))))
```

maxima [A] time = 0.93, size = 112, normalized size = 1.44

$$a^3x-\frac{3\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2b}{d}+\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)ab^2}{d}-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)b^3}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3*(d*x + c + 1/tan(d*x + c))*a^2*b/d + (3*d*x + 3*c + (3*tan(d*x +
c)^2 - 1)/tan(d*x + c)^3)*a*b^2/d - 1/15*(15*d*x + 15*c + (15*tan(d*x + c)^
4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*b^3/d
```

mupad [B] time = 0.45, size = 76, normalized size = 0.97

$$x\,(a-b)^3-\frac{b^3\cot\left(c+d\,x\right)^5}{5\,d}-\frac{\cot\left(c+d\,x\right)^3\,\left(3\,a\,b^2-b^3\right)}{3\,d}-\frac{b\cot\left(c+d\,x\right)\,\left(3\,a^2-3\,a\,b+b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^3,x)
```

```
[Out] x*(a - b)^3 - (b^3*cot(c + d*x)^5)/(5*d) - (cot(c + d*x)^3*(3*a*b^2 - b^3))
/(3*d) - (b*cot(c + d*x)*(3*a^2 - 3*a*b + b^2))/d
```

sympy [A] time = 0.55, size = 126, normalized size = 1.62

$$\begin{cases} a^3x-3a^2bx-\frac{3a^2b\cot\left(c+dx\right)}{d}+3ab^2x-\frac{ab^2\cot^3\left(c+dx\right)}{d}+\frac{3ab^2\cot\left(c+dx\right)}{d}-b^3x-\frac{b^3\cot^5\left(c+dx\right)}{5d}+\frac{b^3\cot^3\left(c+dx\right)}{3d}-\frac{b^3\cot\left(c+dx\right)}{d} \\ x\left(a+b\cot^2\left(c\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*x - 3*a**2*b*cot(c + d*x)/d + 3*a*b**2*x - a*b
**2*cot(c + d*x)**3/d + 3*a*b**2*cot(c + d*x)/d - b**3*x - b**3*cot(c + d*x
)**5/(5*d) + b**3*cot(c + d*x)**3/(3*d) - b**3*cot(c + d*x)/d, Ne(d, 0)), (
x*(a + b*cot(c)**2)**3, True))
```

3.5 $\int \frac{1}{a+b\cot^2(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}+\frac{x}{a-b}$$

```
[Out] x/(a-b)+arctan(cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/(a-b)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}+\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-1),x]
```

```
[Out] x/(a - b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a - b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cot^2(c+dx)}\,dx &= \frac{x}{a-b}-\frac{b\int \frac{\csc^2(c+dx)}{a+b\cot^2(c+dx)}\,dx}{a-b}\\ &= \frac{x}{a-b}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cot(c+dx)\right)}{(a-b)d}\\ &= \frac{x}{a-b}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 1.00

$$\frac{\tan^{-1}(\tan(c+dx)) - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}}{ad-bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[c + d*x]] - (Sqrt[b]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[b]])/Sq
rt[a])/(a*d - b*d)
```

fricas [A] time = 0.46, size = 252, normalized size = 5.14

$$\left[\frac{4\,dx - \sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(2\,dx+2\,c)^2+4\left(a^2-ab-(a^2+ab)\cos(2\,dx+2\,c)\right)\sqrt{-\frac{b}{a}}\sin(2\,dx+2\,c)+a^2-6\,ab+b^2-2\,(a^2-b^2)\cos(2\,dx+2\,c)}{(a^2-2\,ab+b^2)\cos(2\,dx+2\,c)^2+a^2+2\,ab+b^2-2\,(a^2-b^2)\cos(2\,dx+2\,c)}\right)}{4\,(a-b)d}, \; 2 \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*d*x - sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(2*d*x + 2*c)^2 + 4*(a
^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c))*sqrt(-b/a)*sin(2*d*x + 2*c) + a^2
- 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))/((a^2 - 2*a*b + b^2)*cos(2*
d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))))/((a -
b)*d), 1/2*(2*d*x + sqrt(b/a)*arctan(1/2*((a + b)*cos(2*d*x + 2*c) - a + b)
*sqrt(b/a)/(b*sin(2*d*x + 2*c))))/((a - b)*d)]
```

giac [A] time = 0.19, size = 65, normalized size = 1.33

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)} - \frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan(a*tan(d*x + c)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b))/d
```

maple [A] time = 0.33, size = 64, normalized size = 1.31

$$\frac{b\arctan\left(\frac{\cot(dx+c)b}{\sqrt{ab}}\right)}{d\,(a-b)\sqrt{ab}} - \frac{\pi}{2d\,(a-b)} + \frac{\mathrm{arccot}\,(\cot\,(dx+c))}{d\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2),x)
```

```
[Out] 1/d*b/(a-b)/(a*b)^(1/2)*arctan(cot(d*x+c)*b/(a*b)^(1/2))-1/2/d/(a-b)*Pi+1/d
/(a-b)*arccot(cot(d*x+c))
```

maxima [A] time = 0.89, size = 48, normalized size = 0.98

$$-\frac{\frac{b\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)} - \frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(a*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b
))/d
```

mupad [B] time = 0.12, size = 41, normalized size = 0.84

$$\frac{x}{a-b}+\frac{b\,\mathrm{atan}\left(\frac{b\cot(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2),x)
```

```
[Out] x/(a - b) + (b*atan((b*cot(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a - b))
```

sympy [A] time = 1.46, size = 279, normalized size = 5.69

$$\begin{cases} \frac{\tilde{\infty}x}{\cot^2(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{-x+\frac{1}{d\cot(c+dx)}}{b} & \text{for } a=0 \\ \frac{dx\cot^2(c+dx)}{2bd\cot^2(c+dx)+2bd}+\frac{dx}{2bd\cot^2(c+dx)+2bd}-\frac{\cot(c+dx)}{2bd\cot^2(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\cot^2(c)} & \text{for } d=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2i\sqrt{a}\,dx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}d\sqrt{\frac{1}{b}}-2i\sqrt{a}\,bd\sqrt{\frac{1}{b}}}+\frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\cot(c+dx)\right)}{2ia^{\frac{3}{2}}d\sqrt{\frac{1}{b}}-2i\sqrt{a}\,bd\sqrt{\frac{1}{b}}}-\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\cot(c+dx)\right)}{2ia^{\frac{3}{2}}d\sqrt{\frac{1}{b}}-2i\sqrt{a}\,bd\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/cot(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x + 1/(d*co
t(c + d*x)))/b, Eq(a, 0)), (d*x*cot(c + d*x)**2/(2*b*d*cot(c + d*x)**2 + 2*
b*d) + d*x/(2*b*d*cot(c + d*x)**2 + 2*b*d) - cot(c + d*x)/(2*b*d*cot(c + d*
x)**2 + 2*b*d), Eq(a, b)), (x/(a + b*cot(c)**2), Eq(d, 0)), (x/a, Eq(b, 0))
, (2*I*sqrt(a)*d*x*sqrt(1/b)/(2*I*a**(3/2)*d*sqrt(1/b) - 2*I*sqrt(a)*b*d*sq
rt(1/b)) + log(-I*sqrt(a)*sqrt(1/b) + cot(c + d*x))/(2*I*a**(3/2)*d*sqrt(1/
b) - 2*I*sqrt(a)*b*d*sqrt(1/b)) - log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))/(
2*I*a**(3/2)*d*sqrt(1/b) - 2*I*sqrt(a)*b*d*sqrt(1/b)), True))
```

3.6 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}+\frac{b\cot(c+dx)}{2ad(a-b)\left(a+b\cot^2(c+dx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a-b)^2+1/2*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)+1/2*(3*a-b)*arctan(
cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a-b)^2/d
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3661, 414, 522, 203, 205}

$$\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}+\frac{b\cot(c+dx)}{2ad(a-b)\left(a+b\cot^2(c+dx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 + ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*d) + (b*Cot[c + d*x])/(2*a*(a - b)*d*(a + b*Cot[c + d*x]
^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{2a(a-b)d\left(a+b\cot^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \cot(c+dx)\right)}{2a(a-b)d}$$

$$= \frac{b\cot(c+dx)}{2a(a-b)d\left(a+b\cot^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(c+dx)\right)}{(a-b)^2 d} + \frac{((3a-b)b}{}$$

$$= \frac{x}{(a-b)^2} + \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2 d} + \frac{b\cot(c+dx)}{2a(a-b)d\left(a+b\cot^2(c+dx)\right)}$$

Mathematica [A] time = 0.91, size = 90, normalized size = 0.93

$$\frac{\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(a-b)\cot(c+dx)}{a\left(a+b\cot^2(c+dx)\right)} - 2\tan^{-1}(\cot(c+dx))}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-2),x]
```

```
[Out] (-2*ArcTan[Cot[c + d*x]] + ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x])
/Sqrt[a]])/a^(3/2) + ((a - b)*b*Cot[c + d*x])/(a*(a + b*Cot[c + d*x]^2)))/(
2*(a - b)^2*d)
```

fricas [B] time = 0.51, size = 534, normalized size = 5.51

$$\left[\frac{8\left(a^2-ab\right)dx\cos(2dx+2c) - 8\left(a^2+ab\right)dx + \left(3a^2+2ab-b^2-\left(3a^2-4ab+b^2\right)\cos(2dx+2c)\right)\sqrt{-\frac{b}{a}}\,\text{lo}}{8\left(\left(a^4-3a^3b+3a^2b^2-ab^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 - a*b)*d*x*cos(2*d*x + 2*c) - 8*(a^2 + a*b)*d*x + (3*a^2 + 2*a
*b - b^2 - (3*a^2 - 4*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(-b/a)*log(((a^2 + 6
*a*b + b^2)*cos(2*d*x + 2*c)^2 + 4*(a^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c
))*sqrt(-b/a)*sin(2*d*x + 2*c) + a^2 - 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*
x + 2*c))/((a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(
a^2 - b^2)*cos(2*d*x + 2*c))) - 4*(a*b - b^2)*sin(2*d*x + 2*c))/((a^4 - 3*a
^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^4 - a^3*b - a^2*b^2 + a*b
^3)*d), 1/4*(4*(a^2 - a*b)*d*x*cos(2*d*x + 2*c) - 4*(a^2 + a*b)*d*x - (3*a^
2 + 2*a*b - b^2 - (3*a^2 - 4*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(b/a)*arctan(
```

```
1/2*((a + b)*cos(2*d*x + 2*c) - a + b)*sqrt(b/a)/(b*sin(2*d*x + 2*c))) - 2*
(a*b - b^2)*sin(2*d*x + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*
d*x + 2*c) - (a^4 - a^3*b - a^2*b^2 + a*b^3)*d)]
```

giac [A] time = 0.27, size = 123, normalized size = 1.27

$$-\frac{\frac{\left(\pi \left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}-\frac{b\tan(dx+c)}{(a\tan(dx+c)^2+b)(a^2-ab)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan(a*tan(d*x + c)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 -
2*a*b + b^2) - b*tan(d*x + c)/((a*tan(d*x + c)^2 + b)*(a^2 - a*b)))/d
```

maple [B] time = 0.34, size = 173, normalized size = 1.78

$$\frac{b\cot(dx+c)}{2d(a-b)^2\left(a+b\left(\cot^2(dx+c)\right)\right)}-\frac{b^2\cot(dx+c)}{2d(a-b)^2a\left(a+b\left(\cot^2(dx+c)\right)\right)}+\frac{3b\arctan\left(\frac{\cot(dx+c)b}{\sqrt{ab}}\right)}{2d(a-b)^2\sqrt{ab}}-\frac{b^2\arctan\left(\frac{\cot(dx+c}{\sqrt{ab}}\right.}{2d(a-b)^2a\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*b/(a-b)^2*cot(d*x+c)/(a+b*cot(d*x+c)^2)-1/2/d*b^2/(a-b)^2/a*cot(d*x+c
)/(a+b*cot(d*x+c)^2)+3/2/d*b/(a-b)^2/(a*b)^(1/2)*arctan(cot(d*x+c)*b/(a*b)^
(1/2))-1/2/d*b^2/(a-b)^2/a/(a*b)^(1/2)*arctan(cot(d*x+c)*b/(a*b)^(1/2))-1/2
/d/(a-b)^2*Pi+1/d/(a-b)^2*arccot(cot(d*x+c))
```

maxima [A] time = 0.99, size = 115, normalized size = 1.19

$$\frac{\frac{b\tan(dx+c)}{a^2b-ab^2+(a^3-a^2b)\tan(dx+c)^2}-\frac{(3\,ab-b^2)\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}+\frac{2\,(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c)/(a^2*b - a*b^2 + (a^3 - a^2*b)*tan(d*x + c)^2) - (3*a*b
 - b^2)*arctan(a*tan(d*x + c)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)
) + 2*(d*x + c)/(a^2 - 2*a*b + b^2))/d
```

mupad [B] time = 0.79, size = 119, normalized size = 1.23

$$\frac{\frac{a\,x}{(a-b)^2}+\frac{b\,x\cot(c+d\,x)^2}{(a-b)^2}+\frac{b\cot(c+d\,x)}{2\,a\,d\,(a-b)}}{b\cot\left(c+d\,x\right)^2+a}+\frac{\mathrm{atan}\left(\frac{b\cot(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(3\,a\,b-b^2\right)}{\sqrt{a\,b}\,\left(2\,a^3\,d-a\,b\,\left(4\,a\,d-2\,b\,d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(a - b)^2 + (b*x*cot(c + d*x)^2)/(a - b)^2 + (b*cot(c + d*x))/(2*a*d
*(a - b)))/(a + b*cot(c + d*x)^2) + (atan((b*cot(c + d*x))/(a*b)^(1/2))*(3*
a*b - b^2))/((a*b)^(1/2)*(2*a^3*d - a*b*(4*a*d - 2*b*d)))
```

<u>**sympy**</u> **[A]** time = 18.67, size = 2322, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/cot(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - 1/(d*cot
(c + d*x)) + 1/(3*d*cot(c + d*x)**3))/b**2, Eq(a, 0)), (3*d*x*cot(c + d*x)*
*4/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*cot(c + d*x)**2 + 8*b**2*d) + 6*d*
x*cot(c + d*x)**2/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*cot(c + d*x)**2 + 8
*b**2*d) + 3*d*x/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*cot(c + d*x)**2 + 8*
b**2*d) - 3*cot(c + d*x)**3/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*cot(c + d
*x)**2 + 8*b**2*d) - 5*cot(c + d*x)/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*c
ot(c + d*x)**2 + 8*b**2*d), Eq(a, b)), (x/(a + b*cot(c)**2)**2, Eq(d, 0)),
(x/a**2, Eq(b, 0)), (4*I*a**(5/2)*d*x*sqrt(1/b)/(4*I*a**(9/2)*d*sqrt(1/b) +
 4*I*a**(7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) -
8*I*a**(5/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/
b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2) + 4*I*a**(3/2)*b*d*x*sq
rt(1/b)*cot(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1
/b)*cot(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt
(1/b)*cot(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d
*sqrt(1/b)*cot(c + d*x)**2) + 2*I*a**(3/2)*b*sqrt(1/b)*cot(c + d*x)/(4*I*a*
*(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a**(7
/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*a**
(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2) - 2
*I*sqrt(a)*b**2*sqrt(1/b)*cot(c + d*x)/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(
7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5
/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*
a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b
) + cot(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*co
t(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*
cot(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(
1/b)*cot(c + d*x)**2) - 3*a**2*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))/(4*I
*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a*
*(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*
a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2)
+ 3*a*b*log(-I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**2/(4*I*a**(9
/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a**(7/2)
*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*a**(5/
2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2) - a*b*
log(-I*sqrt(a)*sqrt(1/b) + cot(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a*
*(7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**
(5/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*
I*a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2) - 3*a*b*log(I*sqrt(a)*sqrt(1/b
) + cot(c + d*x))*cot(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*
b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b
**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3
/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2) + a*b*log(I*sqrt(a)*sqrt(1/b) + cot(c
 + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*cot(c + d*x
)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*cot(c + d
*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*cot(
c + d*x)**2) - b**2*log(-I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**
2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 -
8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2
+ 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x
)**2) + b**2*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**2/(4*I*a
**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 8*I*a**(
7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 4*I*a*
```

```
*(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2), T
rue))
```

3.7 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=150

$$\frac{b(7a-3b)\cot(c+dx)}{8a^2d(a-b)^2\left(a+b\cot^2(c+dx)\right)}+\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}+\frac{b\cot(c+dx)}{4ad(a-b)\left(a+b\cot^2(c+dx)\right)}$$

```
[Out] x/(a-b)^3+1/4*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^2+1/8*(7*a-3*b)*b*c
ot(d*x+c)/a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)+1/8*(15*a^2-10*a*b+3*b^2)*arctan
(cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)/(a-b)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 203, 205}

$$\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}+\frac{b(7a-3b)\cot(c+dx)}{8a^2d(a-b)^2\left(a+b\cot^2(c+dx)\right)}+\frac{b\cot(c+dx)}{4ad(a-b)\left(a+b\cot^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Cot[c + d*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) + (b*Cot[c + d*x])/(4*a*(a - b)*d*(a
+ b*Cot[c + d*x]^2)^2) + ((7*a - 3*b)*b*Cot[c + d*x])/(8*a^2*(a - b)^2*d*(a
 + b*Cot[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{4a(a-b)d\left(a+b\cot^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \cot(c+dx)\right)}{4a(a-b)d}$$

$$= \frac{b\cot(c+dx)}{4a(a-b)d\left(a+b\cot^2(c+dx)\right)^2} + \frac{(7a-3b)b\cot(c+dx)}{8a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)} - \frac{\operatorname{Subst}\Big(\int \frac{8a^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{b\cot(c+dx)}{4a(a-b)d\left(a+b\cot^2(c+dx)\right)^2} + \frac{(7a-3b)b\cot(c+dx)}{8a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)} - \frac{\operatorname{Subst}\Big(\int \frac{1}{1+\cdots}}{\cdots}$$

$$= \frac{x}{(a-b)^3} + \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} + \frac{b\cot(c+dx)}{4a(a-b)d\left(a+b\cot^2(c+\cdots\right.}$$

Mathematica [A] time = 0.30, size = 138, normalized size = 0.92

$$\frac{\frac{b(7a-3b)(a-b)\cot(c+dx)}{a^2\left(a+b\cot^2(c+dx)\right)} + \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2b(a-b)^2\cot(c+dx)}{a\left(a+b\cot^2(c+dx)\right)^2} - 8\tan^{-1}(\cot(c+dx))}{8d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-3),x]
```

```
[Out] (-8*ArcTan[Cot[c + d*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[
b]*Cot[c + d*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Cot[c + d*x])/(a*(a + b
*Cot[c + d*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Cot[c + d*x])/(a^2*(a + b*Cot[
c + d*x]^2)))/(8*(a - b)^3*d)
```

fricas [B] time = 0.56, size = 1068, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 - 2*a^3*b + a^2*b^2)*d*x*cos(2*d*x + 2*c)^2 - 64*(a^4 - a^2*
b^2)*d*x*cos(2*d*x + 2*c) + 32*(a^4 + 2*a^3*b + a^2*b^2)*d*x - (15*a^4 + 20
*a^3*b - 2*a^2*b^2 - 4*a*b^3 + 3*b^4 + (15*a^4 - 40*a^3*b + 38*a^2*b^2 - 16
*a*b^3 + 3*b^4)*cos(2*d*x + 2*c)^2 - 2*(15*a^4 - 10*a^3*b - 12*a^2*b^2 + 10
*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(2
*d*x + 2*c)^2 + 4*(a^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c))*sqrt(-b/a)*sin
(2*d*x + 2*c) + a^2 - 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))/((a^2 -
 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*
d*x + 2*c))) + 4*(9*a^3*b - 7*a^2*b^2 - 5*a*b^3 + 3*b^4 - 3*(3*a^3*b - 7*a^
2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x + 2*c))*sin(2*d*x + 2*c))/((a^7 - 5*a^6*b
+ 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(2*d*x + 2*c)^2 - 2*(
a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a^3*b^4 + a^2*b^5)*d*cos(2*d*x +
2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5)*d), 1/16*(
16*(a^4 - 2*a^3*b + a^2*b^2)*d*x*cos(2*d*x + 2*c)^2 - 32*(a^4 - a^2*b^2)*d*
x*cos(2*d*x + 2*c) + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*x + (15*a^4 + 20*a^3*b
- 2*a^2*b^2 - 4*a*b^3 + 3*b^4 + (15*a^4 - 40*a^3*b + 38*a^2*b^2 - 16*a*b^3
+ 3*b^4)*cos(2*d*x + 2*c)^2 - 2*(15*a^4 - 10*a^3*b - 12*a^2*b^2 + 10*a*b^3
- 3*b^4)*cos(2*d*x + 2*c))*sqrt(b/a)*arctan(1/2*((a + b)*cos(2*d*x + 2*c) -
 a + b)*sqrt(b/a)/(b*sin(2*d*x + 2*c))) + 2*(9*a^3*b - 7*a^2*b^2 - 5*a*b^3
+ 3*b^4 - 3*(3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x + 2*c))*sin(2*d
*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)
*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a^3*b^
4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^
3*b^4 - a^2*b^5)*d)]
```

giac [A] time = 0.71, size = 206, normalized size = 1.37

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{9\,a^2b\tan(dx+c)^3-5\,ab^2\tan(dx+c)^3+7\,ab^2\tan(dx+c)-3}{\left(a^4-2\,a^3b+a^2b^2\right)\left(a\tan(dx+c)^2+b\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) +
arctan(a*tan(d*x + c)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (9*a^2*b*tan(d*x +
 c)^3 - 5*a*b^2*tan(d*x + c)^3 + 7*a*b^2*tan(d*x + c) - 3*b^3*tan(d*x + c))
/((a^4 - 2*a^3*b + a^2*b^2)*(a*tan(d*x + c)^2 + b)^2))/d
```

maple [B] time = 0.35, size = 363, normalized size = 2.42

$$\frac{7b^2\left(\cot^3(dx+c)\right)}{8d\,(a-b)^3\left(a+b\left(\cot^2(dx+c)\right)\right)^2}-\frac{5b^3\left(\cot^3(dx+c)\right)}{4d\,(a-b)^3\left(a+b\left(\cot^2(dx+c)\right)\right)^2a}+\frac{3b^4\left(\cot^3(dx+c)\right)}{8d\,(a-b)^3\left(a+b\left(\cot^2(dx+c)\right)\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^3,x)
```

```
[Out] 7/8/d*b^2/(a-b)^3/(a+b*cot(d*x+c)^2)^2*cot(d*x+c)^3-5/4/d*b^3/(a-b)^3/(a+b*
cot(d*x+c)^2)^2/a*cot(d*x+c)^3+3/8/d*b^4/(a-b)^3/(a+b*cot(d*x+c)^2)^2/a^2*c
ot(d*x+c)^3+9/8/d*b/(a-b)^3/(a+b*cot(d*x+c)^2)^2*a*cot(d*x+c)-7/4/d*b^2/(a-
b)^3/(a+b*cot(d*x+c)^2)^2*cot(d*x+c)+5/8/d*b^3/(a-b)^3/(a+b*cot(d*x+c)^2)^2
/a*cot(d*x+c)+15/8/d*b/(a-b)^3/(a*b)^(1/2)*arctan(cot(d*x+c)*b/(a*b)^(1/2))
-5/4/d*b^2/(a-b)^3/a/(a*b)^(1/2)*arctan(cot(d*x+c)*b/(a*b)^(1/2))+3/8/d*b^3
```

```
/(a-b)^3/a^2/(a*b)^(1/2)*arctan(cot(d*x+c)*b/(a*b)^(1/2))-1/2/d/(a-b)^3*Pi+
1/d/(a-b)^3*arccot(cot(d*x+c))
```

maxima [A] time = 1.64, size = 228, normalized size = 1.52

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{\left(9\,a^2b-5\,ab^2\right)\tan(dx+c)^3+\left(7\,ab^2-3\,b^3\right)\tan(dx+c)}{a^4b^2-2\,a^3b^3+a^2b^4+\left(a^6-2\,a^5b+a^4b^2\right)\tan(dx+c)^4+2\left(a^5b-2\,a^4b^2+a^3b^3\right)\tan(dx+c)^2}-\frac{8\,(dx+c)}{a^3-3\,a^2b+3\,ab^2-}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(a*tan(d*x + c)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) - ((9*a^2*b - 5*a*b^2)*tan(d*x
+ c)^3 + (7*a*b^2 - 3*b^3)*tan(d*x + c))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 + (
a^6 - 2*a^5*b + a^4*b^2)*tan(d*x + c)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(d*x + c)^2) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

mupad [B] time = 3.29, size = 4866, normalized size = 32.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^3,x)
```

```
[Out] ((cot(c + d*x)^3*(7*a*b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (cot(c +
d*x)*(9*a*b - 5*b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(a^2*d + b^2*d*cot(c + d*x
)^4 + 2*a*b*d*cot(c + d*x)^2) + (2*atan((((((96*a^2*b^10*d^2 - 800*a^3*b^9*
d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5
*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^
3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3
*d^3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2
 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^
2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a*
b^2*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6
*a^6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - (cot(c +
d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8
*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))/(2*a^3*
d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - ((((96*a^2*b^10*d^2 - 800*a^3*b^9*d^
2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d
^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^3
- 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d
^3 + 15*a^8*b^2*d^3)) + (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 +
 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2
- 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a*b^
2*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a
^6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (cot(c + d*
x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d
^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))/(2*a^3*d
- 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d))/((((((96*a^2*b^10*d^2 - 800*a^3*b^9*d^2
 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d^
2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^3 -
 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^
3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 +
2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2 -
 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a*b^2
*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^
6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - (cot(c + d*x
)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d^
```

2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*1i)/(2*a^3 *d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (((((96*a^2*b^10*d^2 - 800*a^3*b^9* d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5 *d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^ 3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3 *d^3 + 15*a^8*b^2*d^3)) + (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^ 2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a* b^2*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6 *a^6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (cot(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 *d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*1i)/(2* a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a^3*b^3)/(32*(a^10*d^3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^3 + 15*a^8*b^2*d^3)))))/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - (atan((((-a^5*b)^(1/2)*((cot(c + d*x)*(9*b^7 - 60* a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d^2 - 4*a^7*b*d^ 2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)) - (((96*a^2*b^10*d^2 - 80 0*a^3*b^9*d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 90 56*a^7*b^5*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(6 4*(a^10*d^3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 + 2304*a^6*b^7*d^2 - 12 80*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2))/(512*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)*(a^8* d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*(-a^5*b) ^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3* a^7*b*d)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2 *d - 3*a^7*b*d)) + ((-a^5*b)^(1/2)*((cot(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a ^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d ^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)) + (((96*a^2*b^10*d^2 - 800*a^3*b^9*d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d^ 2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^ 3 + 15*a^8*b^2*d^3)) + (cot(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^ 2)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^ 2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2* d^2))/(512*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)*(a^8*d^2 - 4*a^7*b *d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*(-a^5*b)^(1/2)*(15*a^ 2 - 10*a*b + 3*b^2))/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)))*(1 5*a^2 - 10*a*b + 3*b^2)*1i)/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b* d)))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a^3*b^3)/(32*(a^10*d^3 - 6*a^9* b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^3 + 15* a^8*b^2*d^3)) - ((-a^5*b)^(1/2)*((cot(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2* b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)) - (((96*a^2*b^10*d^2 - 800*a^3*b^9*d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^3 - 6* a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)* (256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2))/(512*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)*(a^8*d^2 - 4*a^7*b*d^ 2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)))*(15*a ^2 - 10*a*b + 3*b^2))/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)) + ((-a^5*b)^(1/2)*((cot(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^ 4 + 289*a^4*b^3))/(32*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)) + (((96*a^2*b^10*d^2 - 800*a^3*b^9*d^2 + 3040*a^4*b^8*d^2

```
 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d^2 + 5280*a^8*b^4*d^
2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^3 - 6*a^9*b*d^3 + a^4*
b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^3 + 15*a^8*b^2*d^3)
) + (cot(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9*d^2
 - 1280*a^5*b^8*d^2 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^
2 + 2304*a^9*b^4*d^2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2))/(512*(a^8*d -
 a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2
- 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)
)/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)))*(15*a^2 - 10*a*b + 3*
b^2))/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d))))*(-a^5*b)^(1/2)*(
15*a^2 - 10*a*b + 3*b^2)*1i)/(8*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*
d))
```

sympy [A] time = 93.93, size = 9629, normalized size = 64.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/cot(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x + 1/(d*co
t(c + d*x)) - 1/(3*d*cot(c + d*x)**3) + 1/(5*d*cot(c + d*x)**5))/b**3, Eq(a
, 0)), (15*d*x*cot(c + d*x)**6/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(
c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d) + 45*d*x*cot(c + d*x)
**4/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*co
t(c + d*x)**2 + 48*b**3*d) + 45*d*x*cot(c + d*x)**2/(48*b**3*d*cot(c + d*x)
**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d)
+ 15*d*x/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3
*d*cot(c + d*x)**2 + 48*b**3*d) - 15*cot(c + d*x)**5/(48*b**3*d*cot(c + d*x
)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d)
 - 40*cot(c + d*x)**3/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)*
*4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d) - 33*cot(c + d*x)/(48*b**3*d*c
ot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 +
48*b**3*d), Eq(a, b)), (x/(a + b*cot(c)**2)**3, Eq(d, 0)), (x/a**3, Eq(b, 0
)), (16*I*a**(9/2)*d*x*sqrt(1/b)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/
2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(
11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*co
t(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt
(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16
*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)*
*4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*
sqrt(1/b)*cot(c + d*x)**4) + 32*I*a**(7/2)*b*d*x*sqrt(1/b)*cot(c + d*x)**2/
(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2
- 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*
x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b*
*2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(
9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48
*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/
b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) + 18*I
*a**(7/2)*b*sqrt(1/b)*cot(c + d*x)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(1
3/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*
cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sq
rt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 -
16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x
)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*
d*sqrt(1/b)*cot(c + d*x)**4) + 16*I*a**(5/2)*b**2*d*x*sqrt(1/b)*cot(c + d*x
)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x
)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c
 + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/
```

```
2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I
*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sq
rt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) +
 14*I*a**(5/2)*b**2*sqrt(1/b)*cot(c + d*x)**3/(16*I*a**(15/2)*d*sqrt(1/b) +
 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b
) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d
*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2
)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c +
 d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*
cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**
(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) - 28*I*a**(5/2)*b**2*sqrt(1/b)*cot(
c + d*x)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c +
 d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*c
ot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**
(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 +
96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(
1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*
d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**
4) - 20*I*a**(3/2)*b**3*sqrt(1/b)*cot(c + d*x)**3/(16*I*a**(15/2)*d*sqrt(1/
b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt
(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b*
*2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**
(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot
(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1
/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I
*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) + 10*I*a**(3/2)*b**3*sqrt(1/b)*
cot(c + d*x)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot
(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/
b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I
*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**
4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*s
qrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b
**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*
x)**4) + 6*I*sqrt(a)*b**4*sqrt(1/b)*cot(c + d*x)**3/(16*I*a**(15/2)*d*sqrt(
1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sq
rt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*
b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a
**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*c
ot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt
(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16
*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) + 15*a**4*log(-I*sqrt(a)*sqrt
(1/b) + cot(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt
(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*
d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)*
*2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c
 + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)
*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a
**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*c
ot(c + d*x)**4) - 15*a**4*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))/(16*I*a**
(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a*
*(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 9
6*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt
(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3
*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/
2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c
+ d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) + 30*a**3*b*log
(-I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**2/(16*I*a**(15/2)*d*sqr
t(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*
```

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sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2
)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I
*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)
*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sq
rt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 -
16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) - 10*a**3*b*log(-I*sqrt(a)*
sqrt(1/b) + cot(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*
sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b
**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d
*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*c
ot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(
9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32
*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/
b)*cot(c + d*x)**4) - 30*a**3*b*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot
(c + d*x)**2/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot
(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/
b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I
*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**
4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*s
qrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b
**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*
x)**4) + 10*a**3*b*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))/(16*I*a**(15/2)*
d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)
*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**
(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) -
 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt
(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4
*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)*
*2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) + 15*a**2*b**2*log(-I*
sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1/
b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt
(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b*
*2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**
(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot
(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1
/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I
*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) - 20*a**2*b**2*log(-I*sqrt(a)*s
qrt(1/b) + cot(c + d*x))*cot(c + d*x)**2/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I
*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 1
6*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt
(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**
3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)
**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c
 + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)
*b**5*d*sqrt(1/b)*cot(c + d*x)**4) + 3*a**2*b**2*log(-I*sqrt(a)*sqrt(1/b) +
 cot(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*c
ot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(
1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48
*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)
**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d
*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)
*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c +
d*x)**4) - 15*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x
)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x
)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c
 + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/
2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I
*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sq
```

```
rt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) +
 20*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**2/(16*I
*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*
I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4
 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*
sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*
b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a*
*(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*co
t(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) - 3*a**2*b*
*2*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32
*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) +
 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sq
rt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b
**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*
x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot
(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/
2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) - 10*a*b**3*log(-I*sqrt(a)*sqrt(1/b) +
 cot(c + d*x))*cot(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)
*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11
/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(
c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1
/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I
*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4
 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sq
rt(1/b)*cot(c + d*x)**4) + 6*a*b**3*log(-I*sqrt(a)*sqrt(1/b) + cot(c + d*x)
)*cot(c + d*x)**2/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b
)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sq
rt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 +
 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d
*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**
3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7
/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c
 + d*x)**4) + 10*a*b**3*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x
)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x
)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c
 + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/
2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I
*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sq
rt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) -
 6*a*b**3*log(I*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**2/(16*I*a**
(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a*
*(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 9
6*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt
(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3
*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/
2)*b**4*d*sqrt(1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c
+ d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) + 3*b**4*log(-I
*sqrt(a)*sqrt(1/b) + cot(c + d*x))*cot(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1
/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqr
t(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b
**2*d*sqrt(1/b)*cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a*
*(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*co
t(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(
1/b)*cot(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*
I*a**(5/2)*b**5*d*sqrt(1/b)*cot(c + d*x)**4) - 3*b**4*log(I*sqrt(a)*sqrt(1/
b) + cot(c + d*x))*cot(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(1
3/2)*b*d*sqrt(1/b)*cot(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*d*sqrt(1/b)*cot(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*
```

```
cot(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sq
rt(1/b)*cot(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*cot(c + d*x)**2 -
16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x
)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*cot(c + d*x)**2 - 16*I*a**(5/2)*b**5*
d*sqrt(1/b)*cot(c + d*x)**4), True))
```

3.8 $\int \left(1+\cot^2(x)\right)^{3/2} dx$

Optimal. Leaf size=22

$$-\frac{1}{2}\cot(x)\sqrt{\csc^2(x)} - \frac{1}{2}\sinh^{-1}(\cot(x))$$

```
[Out] -1/2*arcsinh(cot(x))-1/2*cot(x)*(csc(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3657, 4122, 195, 215}

$$-\frac{1}{2}\cot(x)\sqrt{\csc^2(x)} - \frac{1}{2}\sinh^{-1}(\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[x]^2)^(3/2),x]
```

```
[Out] -ArcSinh[Cot[x]]/2 - (Cot[x]*Sqrt[Csc[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(1+\cot^2(x)\right)^{3/2} dx &= \int \csc^2(x)^{3/2} dx\\
&= -\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\frac{1}{2}\cot(x)\sqrt{\csc^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \cot(x)\right)\\
&= -\frac{1}{2}\sinh^{-1}(\cot(x)) - \frac{1}{2}\cot(x)\sqrt{\csc^2(x)}
\end{aligned}$$

Mathematica [B] time = 0.10, size = 51, normalized size = 2.32

$$\frac{1}{8}\sin(x)\sqrt{\csc^2(x)}\left(-\csc^2\left(\frac{x}{2}\right)+\sec^2\left(\frac{x}{2}\right)+4\log\left(\sin\left(\frac{x}{2}\right)\right)-4\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[Csc[x]^2]*(-Csc[x/2]^2 - 4*Log[Cos[x/2]] + 4*Log[Sin[x/2]] + Sec[x/2]
^2)*Sin[x])/8
```

fricas [B] time = 0.49, size = 91, normalized size = 4.14

$$-\frac{2\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}(\cos(2x)+1)+\log\left(\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)\sin(2x)-\log\left(-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)\sin(2x)}{4\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*(cos(2*x) + 1) + log(1/2*sqrt(2)*sq
rt(-1/(cos(2*x) - 1))*sin(2*x) + 1)*sin(2*x) - log(-1/2*sqrt(2)*sqrt(-1/(co
s(2*x) - 1))*sin(2*x) + 1)*sin(2*x))/sin(2*x)
```

giac [A] time = 0.18, size = 32, normalized size = 1.45

$$\frac{1}{4}\left(\frac{2\cos(x)}{\cos(x)^2-1}-\log(\cos(x)+1)+\log(-\cos(x)+1)\right)\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*cos(x)/(cos(x)^2 - 1) - log(cos(x) + 1) + log(-cos(x) + 1))*sgn(sin(
x))
```

maple [A] time = 0.27, size = 19, normalized size = 0.86

$$-\frac{\cot(x)\sqrt{1+\cot^2(x)}}{2}-\frac{\operatorname{arcsinh}(\cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(x)^2)^(3/2),x)
```

```
[Out] -1/2*cot(x)*(1+cot(x)^2)^(1/2)-1/2*arcsinh(cot(x))
```

maxima [B] time = 1.10, size = 300, normalized size = 13.64

$$-\frac{4(\cos(3x)+\cos(x))\cos(4x)-4(2\cos(2x)-1)\cos(3x)-8\cos(2x)\cos(x)+\left(2(2\cos(2x)-1)\cos(4x)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(
2*x)*cos(x) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 -
4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x)
```

```
 - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*sin(4
*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))/(2*(2*cos(2*x) -
1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
- 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

mupad [B] time = 0.37, size = 18, normalized size = 0.82

$$-\frac{\operatorname{asinh}\left(\cot(x)\right)}{2}-\frac{\cot(x)\,\sqrt{\cot(x)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 + 1)^(3/2),x)
[Out] - asinh(cot(x))/2 - (cot(x)*(cot(x)^2 + 1)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cot^{2}(x)+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)**2)**(3/2),x)
[Out] Integral((cot(x)**2 + 1)**(3/2), x)
```

3.9 $\int \sqrt{1+\cot^2(x)}\,dx$

Optimal. Leaf size=5

$$-\sinh^{-1}(\cot(x))$$

```
[Out] -arcsinh(cot(x))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3657, 4122, 215}

$$-\sinh^{-1}(\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Cot[x]^2],x]
```

```
[Out] -ArcSinh[Cot[x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\cot^2(x)}\,dx &= \int \sqrt{\csc^2(x)}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \cot(x)\right)\\ &= -\sinh^{-1}(\cot(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 28, normalized size = 5.60

$$\sin(x)\sqrt{\csc^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Cot[x]^2],x]
```

```
[Out] Sqrt[Csc[x]^2]*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x]
```

fricas [B] time = 0.52, size = 53, normalized size = 10.60

$$-\frac{1}{2}\log\left(\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)+\frac{1}{2}\log\left(-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x) + 1) + 1/2*log(-1/2*s
qrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.23, size = 6, normalized size = 1.20

$$-\operatorname{arcsinh}\left(\cot(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(x)^2)^(1/2),x)
```

```
[Out] -arcsinh(cot(x))
```

maxima [B] time = 1.00, size = 35, normalized size = 7.00

$$-\frac{1}{2}\,\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)+\frac{1}{2}\,\log\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1)
```

mupad [B] time = 0.32, size = 5, normalized size = 1.00

$$-\operatorname{asinh}\left(\cot(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 + 1)^(1/2),x)
```

```
[Out] -asinh(cot(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cot^2\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(cot(x)**2 + 1), x)
```

3.10 $\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx$

Optimal. Leaf size=12

$$-\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

```
[Out] -cot(x)/(csc(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3657, 4122, 191}

$$-\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Cot[x]^2],x]
```

```
[Out] -(Cot[x]/Sqrt[Csc[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx &= \int \frac{1}{\sqrt{\csc^2(x)}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx, x, \cot(x)\right)\\ &= -\frac{\cot(x)}{\sqrt{\csc^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Cot[x]^2],x]
[Out] -(Cot[x]/Sqrt[Csc[x]^2])
```

fricas [B] time = 0.43, size = 21, normalized size = 1.75

$$-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.18, size = 13, normalized size = 1.08

$$-\frac{\cot(x)}{\sqrt{1+\cot^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cot(x)^2)^(1/2),x)
[Out] -cot(x)/(1+cot(x)^2)^(1/2)
```

maxima [A] time = 0.83, size = 10, normalized size = 0.83

$$-\frac{1}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/sqrt(tan(x)^2 + 1)
```

mupad [B] time = 0.39, size = 12, normalized size = 1.00

$$-\frac{\sin(2x)}{2\sqrt{\sin(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1)^(1/2),x)
[Out] -sin(2*x)/(2*(sin(x)^2)^(1/2))
```

sympy [A] time = 0.34, size = 14, normalized size = 1.17

$$-\frac{\cot(x)}{\sqrt{\cot^2(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)**2)**(1/2),x)
[Out] -cot(x)/sqrt(cot(x)**2 + 1)
```

3.11 $\int \left(-1-\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}\cot(x)\sqrt{-\csc^2(x)}-\frac{1}{2}\tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

```
[Out] -1/2*arctan(cot(x)/(-csc(x)^2)^(1/2))+1/2*cot(x)*(-csc(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3657, 4122, 195, 217, 203}

$$\frac{1}{2}\cot(x)\sqrt{-\csc^2(x)}-\frac{1}{2}\tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Cot[x]^2)^(3/2),x]
```

```
[Out] -ArcTan[Cot[x]/Sqrt[-Csc[x]^2]]/2 + (Cot[x]*Sqrt[-Csc[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(-1-\cot^2(x)\right)^{3/2} dx = \int \left(-\csc^2(x)\right)^{3/2} dx$$
$$= \text{Subst}\left(\int \sqrt{-1-x^2}\, dx, x, \cot(x)\right)$$
$$= \frac{1}{2}\cot(x)\sqrt{-\csc^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\, dx, x, \cot(x)\right)$$
$$= \frac{1}{2}\cot(x)\sqrt{-\csc^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$
$$= -\frac{1}{2}\tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right) + \frac{1}{2}\cot(x)\sqrt{-\csc^2(x)}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 1.37

$$-\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(-\log\left(\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)\right)+\cot(x)\csc(x)\right)}{4\sqrt{-\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Cot[x]^2)^(3/2),x]
[Out] -1/4*(Csc[x/2]*(Cot[x]*Csc[x] + Log[Cos[x/2]] - Log[Sin[x/2]])*Sec[x/2])/Sq
rt[-Csc[x]^2]
```

fricas [C] time = 0.50, size = 73, normalized size = 2.09

$$\frac{\left(-i\,e^{(4i\,x)}+2i\,e^{(2i\,x)}-i\right)\log\left(e^{(i\,x)}+1\right)+\left(i\,e^{(4i\,x)}-2i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}-1\right)+2i\,e^{(3i\,x)}+2i\,e^{(i\,x)}}{2\left(e^{(4i\,x)}-2\,e^{(2i\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) - 2
*I*e^(2*I*x) + I)*log(e^(I*x) - 1) + 2*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I*x
) - 2*e^(2*I*x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.12, size = 32, normalized size = 0.91

$$\frac{\cot(x)\sqrt{-1-\left(\cot^2(x)\right)}}{2} - \frac{\arctan\left(\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cot(x)^2)^(3/2),x)
[Out] 1/2*cot(x)*(-1-cot(x)^2)^(1/2)-1/2*arctan(cot(x)/(-1-cot(x)^2)^(1/2))
```

maxima [B] time = 1.01, size = 284, normalized size = 8.11

$$\big(2\,(2\,\cos\left(2\,x\right)-1)\cos\left(4\,x\right)-\cos\left(4\,x\right)^{2}-4\,\cos\left(2\,x\right)^{2}-\sin\left(4\,x\right)^{2}+4\,\sin\left(4\,x\right)\sin\left(2\,x\right)-4\,\sin\left(2\,x\right)^{2}+4\,\cos\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
+ 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), cos(
x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*
x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x),
 cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*sin(4
*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*sin(2*x)
 - 4*cos(2*x)*sin(x) + 2*sin(x))/(2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2
- 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*
x) - 1)
```

mupad [B] time = 0.37, size = 31, normalized size = 0.89

$$\frac{\cot(x)\,\sqrt{-\cot(x)^2-1}}{2}-\frac{\mathrm{atan}\left(\frac{\cot(x)}{\sqrt{-\cot(x)^2-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cot(x)^2 - 1)^(3/2),x)
```

```
[Out] (cot(x)*(- cot(x)^2 - 1)^(1/2))/2 - atan(cot(x)/(- cot(x)^2 - 1)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cot^{2}\left(x\right)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)**2)**(3/2),x)
```

```
[Out] Integral((-cot(x)**2 - 1)**(3/2), x)
```

3.12 $\int \sqrt{-1-\cot^2(x)}\,dx$

Optimal. Leaf size=14

$$\tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

```
[Out] arctan(cot(x)/(-csc(x)^2)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4122, 217, 203}

$$\tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Cot[x]^2],x]
```

```
[Out] ArcTan[Cot[x]/Sqrt[-Csc[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-1-\cot^2(x)}\,dx &= \int \sqrt{-\csc^2(x)}\,dx \\
&= \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \cot(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right) \\
&= \tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)
\end{aligned}$$

Mathematica [B] time = 0.02, size = 30, normalized size = 2.14

$$\frac{\csc(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{-\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Cot[x]^2],x]
[Out] (Csc[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]))/Sqrt[-Csc[x]^2]
```

fricas [C] time = 0.57, size = 19, normalized size = 1.36

$$i\log\left(e^{(ix)}+1\right)-i\log\left(e^{(ix)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.13, size = 15, normalized size = 1.07

$$\arctan\left(\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cot(x)^2)^(1/2),x)
[Out] arctan(cot(x)/(-1-cot(x)^2)^(1/2))
```

maxima [A] time = 1.03, size = 17, normalized size = 1.21

$$-\arctan\left(\sin(x),\cos(x)+1\right)+\arctan\left(\sin(x),\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] -arctan2(sin(x), cos(x) + 1) + arctan2(sin(x), cos(x) - 1)
```

mupad [B] time = 0.39, size = 14, normalized size = 1.00

$$\mathrm{atan}\left(\frac{\cot(x)}{\sqrt{-\cot(x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cot(x)^2 - 1)^(1/2),x)
```

```
[Out] atan(cot(x)/(- cot(x)^2 - 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cot^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)**2)**(1/2),x)
[Out] Integral(sqrt(-cot(x)**2 - 1), x)
```

3.13 $\int \frac{1}{\sqrt{-1-\cot^2(x)}} \, dx$

Optimal. Leaf size=14

$$-\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

[Out] -cot(x)/(-csc(x)^2)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3657, 4122, 191}

$$-\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 - Cot[x]^2],x]

[Out] -(Cot[x]/Sqrt[-Csc[x]^2])

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-\cot^2(x)}} \, dx &= \int \frac{1}{\sqrt{-\csc^2(x)}} \, dx \\
&= \text{Subst}\left(\int \frac{1}{\left(-1-x^2\right)^{3/2}} \, dx, x, \cot(x)\right) \\
&= -\frac{\cot(x)}{\sqrt{-\csc^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Cot[x]^2],x]
[Out] -(Cot[x]/Sqrt[-Csc[x]^2])
```

fricas [C] time = 0.84, size = 14, normalized size = 1.00

$$\frac{1}{2}\left(-i\,e^{(2i\,x)}-i\right)e^{(-i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(-I*e^(2*I*x) - I)*e^(-I*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.10, size = 15, normalized size = 1.07

$$-\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-cot(x)^2)^(1/2),x)
[Out] -cot(x)/(-1-cot(x)^2)^(1/2)
```

maxima [A] time = 0.50, size = 12, normalized size = 0.86

$$-\frac{1}{\sqrt{-\tan(x)^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/sqrt(-tan(x)^2 - 1)
```

mupad [B] time = 0.68, size = 13, normalized size = 0.93

$$\frac{\sin\left(2\,x\right)\,1\mathrm{i}}{2\,\sqrt{{\sin(x)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- cot(x)^2 - 1)^(1/2),x)
[Out] (sin(2*x)*1i)/(2*(sin(x)^2)^(1/2))
```

sympy [A] time = 0.34, size = 15, normalized size = 1.07

$$-\frac{\cot\left(x\right)}{\sqrt{-\cot^2\left(x\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)**2)**(1/2),x)
[Out] -cot(x)/sqrt(-cot(x)**2 - 1)
```

3.14 $\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}} \, dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{a\csc^2(x)}}{a}-\frac{1}{\sqrt{a\csc^2(x)}}$$

```
[Out] -1/(a*csc(x)^2)^(1/2)-(a*csc(x)^2)^(1/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$-\frac{\sqrt{a\csc^2(x)}}{a}-\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] -(1/Sqrt[a*Csc[x]^2]) - Sqrt[a*Csc[x]^2]/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}} \, dx &= \int \frac{\cot^3(x)}{\sqrt{a\csc^2(x)}} \, dx \\
&= -\left(\frac{1}{2}a\,\text{Subst}\left(\int \frac{-1+x}{(ax)^{3/2}} \, dx, x, \csc^2(x)\right)\right) \\
&= -\left(\frac{1}{2}a\,\text{Subst}\left(\int \left(-\frac{1}{(ax)^{3/2}}+\frac{1}{a\sqrt{ax}}\right) dx, x, \csc^2(x)\right)\right) \\
&= -\frac{1}{\sqrt{a\csc^2(x)}}-\frac{\sqrt{a\csc^2(x)}}{a}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 19, normalized size = 0.68

$$\frac{-\csc^2(x)-1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/Sqrt[a + a*Cot[x]^2],x]
[Out] (-1 - Csc[x]^2)/Sqrt[a*Csc[x]^2]
```

fricas [A] time = 0.43, size = 27, normalized size = 0.96

$$\frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}\,(\cos(2x)-3)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*sqrt(-a/(cos(2*x) - 1))*(cos(2*x) - 3)/a
```

giac [A] time = 0.18, size = 30, normalized size = 1.07

$$-\frac{\sqrt{-a\cos(x)^2+a}+\frac{a}{\sqrt{-a\cos(x)^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
[Out] -(sqrt(-a*cos(x)^2 + a) + a/sqrt(-a*cos(x)^2 + a))/a
```

maple [A] time = 0.30, size = 29, normalized size = 1.04

$$-\frac{\sqrt{a+a\left(\cot^2(x)\right)}}{a}-\frac{1}{\sqrt{a+a\left(\cot^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+a*cot(x)^2)^(1/2),x)
[Out] -1/a*(a+a*cot(x)^2)^(1/2)-1/(a+a*cot(x)^2)^(1/2)
```

maxima [A] time = 0.74, size = 24, normalized size = 0.86

$$-\frac{1}{\sqrt{\frac{a}{\sin(x)^2}}}-\frac{\sqrt{\frac{a}{\sin(x)^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/sqrt(a/sin(x)^2) - sqrt(a/sin(x)^2)/a
```

mupad [B] time = 0.59, size = 17, normalized size = 0.61

$$-\frac{\sin(x)^2+1}{\sqrt{a}\,\sqrt{\sin(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + a*cot(x)^2)^(1/2),x)
```

```
[Out] -(sin(x)^2 + 1)/(a^(1/2)*(sin(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(x)}{\sqrt{a\left(\cot^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**3/sqrt(a*(cot(x)**2 + 1)), x)
```

3.15 $\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}-\frac{\csc(x)\tanh^{-1}(\cos(x))}{\sqrt{a\csc^2(x)}}$$

```
[Out] cot(x)/(a*csc(x)^2)^(1/2)-arctanh(cos(x))*csc(x)/(a*csc(x)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4125, 2592, 321, 206}

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}-\frac{\csc(x)\tanh^{-1}(\cos(x))}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] Cot[x]/Sqrt[a*Csc[x]^2] - (ArcTanh[Cos[x]]*Csc[x])/Sqrt[a*Csc[x]^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \int \frac{\cot^2(x)}{\sqrt{a\csc^2(x)}}\,dx$$
$$= \frac{\csc(x)\int \cos(x)\cot(x)\,dx}{\sqrt{a\csc^2(x)}}$$
$$= -\frac{\csc(x)\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right)}{\sqrt{a\csc^2(x)}}$$
$$= \frac{\cot(x)}{\sqrt{a\csc^2(x)}} - \frac{\csc(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)}{\sqrt{a\csc^2(x)}}$$
$$= \frac{\cot(x)}{\sqrt{a\csc^2(x)}} - \frac{\tanh^{-1}(\cos(x))\csc(x)}{\sqrt{a\csc^2(x)}}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.03

$$\frac{\csc(x)\left(\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[a + a*Cot[x]^2],x]
[Out] (Csc[x]*(Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]))/Sqrt[a*Csc[x]^2]
```

fricas [B] time = 0.75, size = 77, normalized size = 2.48

$$\frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}\,\sin(2x)+\sqrt{a}\,\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{-\frac{a}{\cos(2x)-1}}\,\sin(2x)-a\cos(2x)-3a}{\cos(2x)-1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*sqrt(-a/(cos(2*x) - 1))*sin(2*x) + sqrt(a)*log((2*sqrt(2)*sqrt
(a)*sqrt(-a/(cos(2*x) - 1))*sin(2*x) - a*cos(2*x) - 3*a)/(cos(2*x) - 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.20, size = 38, normalized size = 1.23

$$-\frac{\ln\left(\sqrt{a}\,\cot(x)+\sqrt{a+a\left(\cot^2(x)\right)}\right)}{\sqrt{a}}+\frac{\cot(x)}{\sqrt{a+a\left(\cot^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*cot(x)^2)^(1/2),x)
```

```
[Out] -ln(a^(1/2)*cot(x)+(a+a*cot(x)^2)^(1/2))/a^(1/2)+cot(x)/(a+a*cot(x)^2)^(1/2
)
```

maxima [A] time = 0.77, size = 27, normalized size = 0.87

$$-\frac{\sqrt{-a}\left(\arctan\left(\sin(x),\cos(x)+1\right)-\arctan\left(\sin(x),\cos(x)-1\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(x)^2}{\sqrt{a\cot(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + a*cot(x)^2)^(1/2),x)
```

```
[Out] int(cot(x)^2/(a + a*cot(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\sqrt{a\left(\cot^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**2/sqrt(a*(cot(x)**2 + 1)), x)
```

3.16 $\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{\sqrt{a\csc^2(x)}}$$

```
[Out] 1/(a*csc(x)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3657, 4124, 32}

$$\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] 1/Sqrt[a*Csc[x]^2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx &= \int \frac{\cot(x)}{\sqrt{a\csc^2(x)}}\,dx\\ &= -\left(\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \csc^2(x)\right)\right)\\ &= \frac{1}{\sqrt{a\csc^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + a*Cot[x]^2],x]
[Out] 1/Sqrt[a*Csc[x]^2]
```

fricas [B] time = 0.52, size = 27, normalized size = 2.70

$$-\frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}\left(\cos(2x)-1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*sqrt(-a/(cos(2*x) - 1))*(cos(2*x) - 1)/a
```

giac [A] time = 0.20, size = 12, normalized size = 1.20

$$\frac{\sqrt{a\sin(x)^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
[Out] sqrt(a*sin(x)^2)/a
```

maple [A] time = 0.19, size = 11, normalized size = 1.10

$$\frac{1}{\sqrt{a+a\left(\cot^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*cot(x)^2)^(1/2),x)
[Out] 1/(a+a*cot(x)^2)^(1/2)
```

maxima [A] time = 0.55, size = 8, normalized size = 0.80

$$\frac{1}{\sqrt{\frac{a}{\sin(x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/sqrt(a/sin(x)^2)
```

mupad [B] time = 0.48, size = 10, normalized size = 1.00

$$\frac{\sqrt{\sin(x)^2}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + a*cot(x)^2)^(1/2),x)
[Out] (sin(x)^2)^(1/2)/a^(1/2)
```

sympy [A] time = 1.20, size = 12, normalized size = 1.20

$$\frac{1}{\sqrt{a\cot^2(x)+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] 1/sqrt(a*cot(x)**2 + a)
```

3.17 $\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{1}{\sqrt{a\csc^2(x)}}$$

```
[Out] arctanh((a*csc(x)^2)^(1/2)/a^(1/2))/a^(1/2)-1/(a*csc(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4124, 51, 63, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a*Csc[x]^2]/Sqrt[a]]/Sqrt[a] - 1/Sqrt[a*Csc[x]^2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx &= \int \frac{\tan(x)}{\sqrt{a\csc^2(x)}}\,dx\\ &= -\left(\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx,x,\csc^2(x)\right)\right)\\ &= -\frac{1}{\sqrt{a\csc^2(x)}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\csc^2(x)\right)\\ &= -\frac{1}{\sqrt{a\csc^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\csc^2(x)}\right)}{a}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{1}{\sqrt{a\csc^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 1.36

$$-\frac{\csc(x)\left(\sin(x)+\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] -((Csc[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x]))/S
qrt[a*Csc[x]^2])
```

fricas [B] time = 0.44, size = 78, normalized size = 2.17

$$\frac{\left(\tan(x)^2+1\right)\sqrt{a}\,\log\left(2\,a\tan(x)^2+2\,\sqrt{a}\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}\,\tan(x)^2+a\right)-2\,\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}\,\tan(x)^2}{2\left(a\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 +
 a)/tan(x)^2)*tan(x)^2 + a) - 2*sqrt((a*tan(x)^2 + a)/tan(x)^2)*tan(x)^2)/(
a*tan(x)^2 + a)
```

giac [A] time = 0.19, size = 42, normalized size = 1.17

$$-\frac{\arctan\left(\frac{\sqrt{-a\cos(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{-a\cos(x)^2+a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(-a*cos(x)^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(-a*cos(x)^2 + a)/a
```

maple [A] time = 0.74, size = 56, normalized size = 1.56

$$-\frac{\left(\sin(x)+\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right)-\ln\left(-\frac{-\sin(x)-1+\cos(x)}{\sin(x)}\right)\right)\sqrt{4}}{2\sin(x)\sqrt{-\frac{a}{-1+\cos^2(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*cot(x)^2)^(1/2),x)
```

```
[Out] -1/2*(sin(x)+ln(-(-1+cos(x)+sin(x))/sin(x))-ln(-(-sin(x)-1+cos(x))/sin(x)))
/sin(x)/(-1/(-1+cos(x)^2)*a)^(1/2)*4^(1/2)
```

maxima [A] time = 0.65, size = 52, normalized size = 1.44

$$-\frac{1}{2}\,a\left(\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{a}{\sin(x)^2}}}{\sqrt{a}+\sqrt{\frac{a}{\sin(x)^2}}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{\frac{a}{\sin(x)^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(-(sqrt(a) - sqrt(a/sin(x)^2))/(sqrt(a) + sqrt(a/sin(x)^2)))/a^(
3/2) + 2/(a*sqrt(a/sin(x)^2)))
```

mupad [B] time = 0.42, size = 20, normalized size = 0.56

$$\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{\sin(x)^2}}\right)-\sqrt{\sin(x)^2}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + a*cot(x)^2)^(1/2),x)
```

```
[Out] (atanh((1/sin(x)^2)^(1/2)) - (sin(x)^2)^(1/2))/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(x)}{\sqrt{a\left(\cot^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a*(cot(x)**2 + 1)), x)
```

3.18 $\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}+\frac{\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

```
[Out] cot(x)/(a*csc(x)^2)^(1/2)+csc(x)*sec(x)/(a*csc(x)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2590, 14}

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}+\frac{\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] Cot[x]/Sqrt[a*Csc[x]^2] + (Csc[x]*Sec[x])/Sqrt[a*Csc[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx &= \int \frac{\tan^2(x)}{\sqrt{a\csc^2(x)}}\,dx\\ &= \frac{\csc(x)\int \sin(x)\tan^2(x)\,dx}{\sqrt{a\csc^2(x)}}\\ &= -\frac{\csc(x)\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(x)\right)}{\sqrt{a\csc^2(x)}}\\ &= -\frac{\csc(x)\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(x)\right)}{\sqrt{a\csc^2(x)}}\\ &= \frac{\cot(x)}{\sqrt{a\csc^2(x)}}+\frac{\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 19, normalized size = 0.66

$$\frac{\cot(x)+\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + a*Cot[x]^2],x]
[Out] (Cot[x] + Csc[x]*Sec[x])/Sqrt[a*Csc[x]^2]
```

fricas [A] time = 0.41, size = 35, normalized size = 1.21

$$\frac{\left(\tan(x)^3+2\,\tan(x)\right)\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}}{a\tan(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] (tan(x)^3 + 2*tan(x))*sqrt((a*tan(x)^2 + a)/tan(x)^2)/(a*tan(x)^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.57, size = 33, normalized size = 1.14

$$\frac{\left(\sin^3(x)\right)\sqrt{4}}{2\sqrt{-\frac{a}{-1+\cos^2(x)}}\,\cos(x)\left(-1+\cos(x)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*cot(x)^2)^(1/2),x)
[Out] 1/2*sin(x)^3/(-1/(-1+cos(x)^2)*a)^(1/2)/cos(x)/(-1+cos(x))^2*4^(1/2)
```

maxima [A] time = 0.52, size = 18, normalized size = 0.62

$$\frac{\tan(x)^2 + 2}{\sqrt{\tan(x)^2 + 1}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (tan(x)^2 + 2)/(sqrt(tan(x)^2 + 1)*sqrt(a))
```

mupad [B] time = 0.73, size = 34, normalized size = 1.17

$$\frac{\tan(x)^3\,\sqrt{\frac{1}{\tan(x)^2}} + 2\tan(x)\,\sqrt{\frac{1}{\tan(x)^2}}}{\sqrt{a}\,\sqrt{\tan(x)^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + a*cot(x)^2)^(1/2),x)
```

```
[Out] (tan(x)^3*(1/tan(x)^2)^(1/2) + 2*tan(x)*(1/tan(x)^2)^(1/2))/(a^(1/2)*(tan(x
)^2 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\sqrt{a\left(\cot^2(x) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a*(cot(x)**2 + 1)), x)
```

3.19 $\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=66

$$-\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}+\sqrt{a+b\cot^2(x)}-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

```
[Out] -1/3*(a+b*cot(x)^2)^(3/2)/b-arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))*(a-b)
^(1/2)+(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 80, 50, 63, 208}

$$-\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}+\sqrt{a+b\cot^2(x)}-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) + Sqrt[a + b*Cot[x
]^2] - (a + b*Cot[x]^2)^(3/2)/(3*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{1+x}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b} + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \cot^2(x)\right)\\
&= \sqrt{a+b\cot^2(x)} - \frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b} + \frac{1}{2}(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \text{co}\right.\\
&= \sqrt{a+b\cot^2(x)} - \frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(}\right.}{b}\\
&= -\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) + \sqrt{a+b\cot^2(x)} - \frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 65, normalized size = 0.98

$$-\frac{\sqrt{a+b\cot^2(x)}\left(a+b\cot^2(x)-3b\right)}{3b} - \sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) - (Sqrt[a + b*Cot[
x]^2]*(a - 3*b + b*Cot[x]^2))/(3*b)
```

fricas [B] time = 0.72, size = 330, normalized size = 5.00

$$\left[\frac{3\left(b\cos(2x)-b\right)\sqrt{a-b}\,\log\left(-2\left(a^2-2ab+b^2\right)\cos(2x)^2-2a^2+b^2+2\left((a-b)\cos(2x)^2-(2a-b)\cos(2x\right.\right.}{12\left(b\cos(2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cos(2*x) - b)*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2
 - 2*a^2 + b^2 + 2*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b
)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x))
 - 4*((a - 4*b)*cos(2*x) - a + 2*b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*
x) - 1)))/(b*cos(2*x) - b), -1/6*(3*(b*cos(2*x) - b)*sqrt(-a + b)*arctan(-s
qrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1)/
((a - b)*cos(2*x) - a)) + 2*((a - 4*b)*cos(2*x) - a + 2*b)*sqrt(((a - b)*co
s(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x) - b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(x))]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%
%%{2,[0,2,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%
%%{-2,[1,4,1]%%%}+%%%{6,[1,2,1]%%%}+%%%{-4,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%%
%{-4,[0,2,2]%%%}+%%%{4,[0,0,2]%%%}] at parameters values [86,-97,-82]Warnin
g, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%
}+%%%{4,[0,2]%%%}] at parameters values [90.79236355,54.1277311612]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [69.8278764193,63.4443001123]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [108.020125429,82.1195442914]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [26.4357969165,7.79369851155]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [150.357303702,71.707969239]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[b+63,75]Unable to conver
t to real 75.0*(b+63.0)-5625.0 Error: Bad Argument ValueWarning, choosing r
oot of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{2,[0,2,1]%%%},0,%%%{1
,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{-2,[1,4,1]%%%}+%%%{6,
[1,2,1]%%%}+%%%{-4,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%%%{-4,[0,2,2]%%%}+%%%{4,[
0,0,2]%%%}] at parameters values [18,-49,-33]Warning, choosing root of [1,0
,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at pa
rameters values [70.2095400225,15.451549686]Warning, choosing root of [1,0,
%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at par
ameters values [100.356811349,81.9516051291]Warning, choosing root of [1,0,
%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at par
ameters values [133.032670634,51.6443148847]Warning, choosing root of [1,0,
%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at par
ameters values [42.28121641,31.8503101398]Warning, choosing root of [1,0,%%
%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at param
eters values [92.8262473457,64.3995612673]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a,b]=[b+66,40]Unable to convert to real 40.0*(b+66.0)-
1600.0 Error: Bad Argument ValueUnable to cancel step at 0 of 2*(((3*a-6*b)
*sqrt(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^4+6*b^2*sqrt(a-
b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2+(a*b^2-4*b^3)*sqrt(a-
b))/3/((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2-b)^3+sqrt(a-b)/4*
ln((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2))--2*(((3*a-6*b)*sqrt
(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^4+6*b^2*sqrt(a-b)*(s
qrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2+(a*b^2-4*b^3)*sqrt(a-b))/3
/((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2-b)^3+sqrt(a-b)/4*ln((s
```

```
qrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2))Discontinuities at zeroes
 of sin(x) were not checkedEvaluation time: 0.8Done
```

maple [A] time = 0.19, size = 84, normalized size = 1.27

$$-\frac{\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}{3b}+\sqrt{a+b\left(\cot^2(x)\right)}-\frac{b\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}+\frac{a\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -1/3*(a+b*cot(x)^2)^(3/2)/b+(a+b*cot(x)^2)^(1/2)-b/(-a+b)^(1/2)*arctan((a+b
*cot(x)^2)^(1/2)/(-a+b)^(1/2))+a/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(
-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 3.16, size = 66, normalized size = 1.00

$$\sqrt{b\cot(x)^2+a}-\frac{\left(b\cot(x)^2+a\right)^{3/2}}{3\,b}+2\,\mathrm{atan}\left(\frac{2\,\sqrt{b\cot(x)^2+a}\,\sqrt{\frac{b}{4}-\frac{a}{4}}}{a-b}\right)\sqrt{\frac{b}{4}-\frac{a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] (a + b*cot(x)^2)^(1/2) - (a + b*cot(x)^2)^(3/2)/(3*b) + 2*atan((2*(a + b*co
t(x)^2)^(1/2)*(b/4 - a/4)^(1/2))/(a - b))*(b/4 - a/4)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\cot^2(x)}\,\cot^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*cot(x)**3, x)
```

3.20 $\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=48

$$\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\sqrt{a+b\cot^2(x)}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)-(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 50, 63, 208}

$$\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - Sqrt[a + b*Cot[x]^2
]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \cot^2(x)\right)\right)\\ &= -\sqrt{a+b\cot^2(x)} - \frac{1}{2}(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\\ &= -\sqrt{a+b\cot^2(x)} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}\\ &= \sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \sqrt{a+b\cot^2(x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.00

$$\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - Sqrt[a + b*Cot[x]^2
]
```

fricas [B] time = 1.51, size = 248, normalized size = 5.17

$$\left[\frac{1}{4}\sqrt{a-b}\,\log\left(-2\left(a^2-2\,ab+b^2\right)\cos\left(2\,x\right)^2-2\,a^2+b^2-2\left((a-b)\cos\left(2\,x\right)^2-(2\,a-b)\cos\left(2\,x\right)+a\right)\sqrt{a-b}\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 - 2*((
a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*cos(2
*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) - sqrt(((a - b)*cos(
2*x) - a - b)/(cos(2*x) - 1)), 1/2*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1)/((a - b)*cos(2*x)
- a)) - sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))]
```

giac [B] time = 0.80, size = 95, normalized size = 1.98

$$-\frac{1}{2}\left(\sqrt{a-b}\,\log\left(\left(\sqrt{a-b}\,\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)-\frac{4\sqrt{a-b}\,b}{\left(\sqrt{a-b}\,\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(a - b)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 +
b))^2) - 4*sqrt(a - b)*b/((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^
2 + b))^2 - b))*sgn(sin(x))
```

maple [A] time = 0.13, size = 71, normalized size = 1.48

$$-\sqrt{a+b\left(\cot^2(x)\right)}+\frac{b\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{a\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -(a+b*cot(x)^2)^(1/2)+b/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/
2))-a/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 1.17, size = 53, normalized size = 1.10

$$-\sqrt{b\cot(x)^2+a}-2\,\mathrm{atan}\left(\frac{2\sqrt{b\cot(x)^2+a}\,\sqrt{\frac{b}{4}-\frac{a}{4}}}{a-b}\right)\sqrt{\frac{b}{4}-\frac{a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] - (a + b*cot(x)^2)^(1/2) - 2*atan((2*(a + b*cot(x)^2)^(1/2)*(b/4 - a/4)^(1/
2))/(a - b))*(b/4 - a/4)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\cot^2(x)}\,\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*cot(x), x)
```

3.21 $\int \sqrt{a+b\cot^2(x)}\,\tan(x)\,dx$

Optimal. Leaf size=60

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))*a^(1/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))*(a-b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 446, 83, 63, 208}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2]*Tan[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - Sqrt[a - b]*ArcTanh[Sqrt[a
+ b*Cot[x]^2]/Sqrt[a - b]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cot^2(x)}\,\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(1+x^2\right)}\,dx,x,\cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(1+x)}\,dx,x,\cot^2(x)\right)\right)\\
&= -\left(\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\cot^2(x)\right)\right)-\frac{1}{2}(-a+b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+
\end{aligned}
$$

$$
= -\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cot^2(x)}\right)}{b}-\frac{(-a+b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a}\right.}{b}
$$

$$
= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)
$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.00

$$
\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - Sqrt[a - b]*ArcTanh[Sqrt[a
+ b*Cot[x]^2]/Sqrt[a - b]]
```

fricas [A] time = 0.50, size = 351, normalized size = 5.85

$$
\left[\frac{1}{2}\sqrt{a}\log\left(2\,a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)+\frac{1}{2}\sqrt{a-b}\log\left(\frac{(2\,a-b)\tan(x)^2-2\sqrt{a-b}\sqrt{\frac{a\,t}{t}}}{\tan(x)^2+1}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*t
an(x)^2 + b) + 1/2*sqrt(a - b)*log(((2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt
((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)), -sqrt(-a + b)*ar
ctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) + 1/2*sqrt(a)*l
og(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b),
-sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) + 1/2*sqrt(a -
 b)*log(((2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)
*tan(x)^2 + b)/(tan(x)^2 + 1)), -sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2
+ b)/tan(x)^2)/a) - sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)
/tan(x)^2)/(a - b))]
```

giac [B] time = 0.74, size = 187, normalized size = 3.12

$$
\frac{1}{2}\left(\frac{2\sqrt{a-b}\,a\arctan\left(\frac{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2-2\,a+b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}}+\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\,\text{si}\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(a - b)*a*arctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*
sin(x)^2 + b))^2 - 2*a + b)/sqrt(-a^2 + a*b))/sqrt(-a^2 + a*b) + sqrt(a - b
)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2))*sgn(sin(
x)) - 1/2*(2*sqrt(a - b)*a*arctan(-(a - b)/sqrt(-a^2 + a*b)) + sqrt(-a^2 +
a*b)*sqrt(a - b)*log(b))*sgn(sin(x))/sqrt(-a^2 + a*b)
```

maple [C] time = 1.07, size = 591, normalized size = 9.85

$$\frac{\left(\text{EllipticF}\left(\frac{(-1+\cos(x))\sqrt{\frac{2\sqrt{a}\,\sqrt{a-b}-2a+b}{b}}}{\sin(x)}, \sqrt{\frac{8a^{\frac{3}{2}}\sqrt{a-b}-4\sqrt{a}\,\sqrt{a-b}\,b+8a^2-8ab+b^2}{b^2}}\right)b-2\,\text{EllipticPi}\left(\frac{(-1+\cos(x))\sqrt{\frac{2\sqrt{a}\,\sqrt{a-b}-2a+b}{b}}}{\sin(x)},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(1/2)*tan(x),x)
```

```
[Out] 1/2*(EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),(
(8*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))
*b-2*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),
1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/
((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a+2*EllipticPi((-1+cos(x))*((2*a^(
1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,
(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^
(1/2))*a-2*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/s
in(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)
^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*b)*sin(x)^3*(-2*(cos(x)*a^(
1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/
2)*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(
x)+a)/(cos(x)+1)/b)^(1/2)*((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))^(1/2)/(
-1+cos(x))/(a*cos(x)^2-b*cos(x)^2-a)*4^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)
/b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cot(x)^2 + a}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^2 + a)*tan(x), x)
```

mupad [B] time = 0.48, size = 69, normalized size = 1.15

$$\operatorname{atanh}\left(\frac{2\,a\,b^3\,\sqrt{a-b}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{2\,a\,b^4-2\,a^2\,b^3}\right)\sqrt{a-b}+\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan(x)^2}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] atanh((2*a*b^3*(a - b)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a*b^4 - 2*a^2*b^3))
*(a - b)^(1/2) + a^(1/2)*atanh((a + b/tan(x)^2)^(1/2)/a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cot^2(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*tan(x), x)
```

3.22 $\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}+\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{2\sqrt{b}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-1/2*(a-2*b)*arc
tanh(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)-1/2*cot(x)*(a+b*cot(x)^2)
^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3670, 478, 523, 217, 206, 377, 203}

$$-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}+\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - ((a - 2*b)*
ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]])/(2*Sqrt[b]) - (Cot[x]*Sqrt[
a + b*Cot[x]^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= -\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)} + (a-b)\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= -\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)} + (a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2}}\right.\\
&= \sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{2\sqrt{b}} - \frac{1}{2}\cot(x)\sqrt{a
\end{aligned}$$

Mathematica [B] time = 22.40, size = 2105, normalized size = 23.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[x]^2*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -1/2*(Sqrt[(-a - b + a*Cos[2*x] - b*Cos[2*x])/(-1 + Cos[2*x])]*Cot[x]) + ((
-4*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]
^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)
*Cos[x])*Sec[x/2]^2)/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])
] + (a - 2*b)*ArcTanh[(2*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*
Sec[x/2]^4 + 4*a*Tan[x/2]^2])])*((b*Sqrt[-(a/(-1 + Cos[2*x])) - b/(-1 + Cos
[2*x]) + (a*Cos[2*x])/(-1 + Cos[2*x]) - (b*Cos[2*x])/(-1 + Cos[2*x])])/(-a
- b + a*Cos[2*x] - b*Cos[2*x]) - (a*Cos[2*x]*Sqrt[-(a/(-1 + Cos[2*x])) - b/
(-1 + Cos[2*x]) + (a*Cos[2*x])/(-1 + Cos[2*x]) - (b*Cos[2*x])/(-1 + Cos[2*x
])])/(-a - b + a*Cos[2*x] - b*Cos[2*x]) + (b*Cos[2*x]*Sqrt[-(a/(-1 + Cos[2*
x])) - b/(-1 + Cos[2*x]) + (a*Cos[2*x])/(-1 + Cos[2*x]) - (b*Cos[2*x])/(-1
+ Cos[2*x])])/(-a - b + a*Cos[2*x] - b*Cos[2*x]))*Sqrt[a + b*Cot[x]^2]*Tan[
x/2])/(Sqrt[2]*Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]*(((-4*S
```

```
qrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]^2*S
ec[x/2]^4 + 4*a*Tan[x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)*Cos
[x])*Sec[x/2]^2)/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])] +
(a - 2*b)*ArcTanh[(2*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*Sec[
x/2]^4 + 4*a*Tan[x/2]^2])])*Sqrt[a + b*Cot[x]^2*Sec[x/2]^2)/(2*Sqrt[2]*Sqr
t[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]) - (Sqrt[b]*(-4*Sqrt[a -
b]*Sqrt[b]*ArcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]^2*Sec[x/2]^
4 + 4*a*Tan[x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)*Cos[x])*Sec
[x/2]^2)/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])] + (a - 2*b
)*ArcTanh[(2*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*Sec[x/2]^4 +
 4*a*Tan[x/2]^2])])*Cot[x]*Csc[x]^2*Tan[x/2])/(Sqrt[2]*Sqrt[a + b*Cot[x]^2]
*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]) - ((-4*Sqrt[a - b]*Sqrt[b]*A
rcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[
x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)*Cos[x])*Sec[x/2]^2)/(Sq
rt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])] + (a - 2*b)*ArcTanh[(2
*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2
]^2])])*Sqrt[a + b*Cot[x]^2]*Tan[x/2]*(-2*(-a + b)*Sec[x/2]^4*Sin[2*x] + 2*
(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4*Tan[x/2]))/(2*Sqrt[2]*Sqrt[b]*((a +
b + (-a + b)*Cos[2*x])*Sec[x/2]^4)^(3/2)) + (Sqrt[a + b*Cot[x]^2]*Tan[x/2]*
(-(((a - 2*b)*(-((Sqrt[2]*(-a + b)*Sec[x/2]^2*Sin[x])/(Sqrt[b]*Sqrt[(a + b
+ (-a + b)*Cos[2*x])*Sec[x/2]^4])) + (Sqrt[2]*(a + (-a + b)*Cos[x])*Sec[x/2
]^2*Tan[x/2])/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]) - ((a
+ (-a + b)*Cos[x])*Sec[x/2]^2*(-2*(-a + b)*Sec[x/2]^4*Sin[2*x] + 2*(a + b +
 (-a + b)*Cos[2*x])*Sec[x/2]^4*Tan[x/2]))/(Sqrt[2]*Sqrt[b]*((a + b + (-a +
b)*Cos[2*x])*Sec[x/2]^4)^(3/2))))/(1 - (2*(a + (-a + b)*Cos[x])^2)/(b*(a +
b + (-a + b)*Cos[2*x])))) - (4*Sqrt[a - b]*Sqrt[b]*(-1/2*(Sqrt[a - b]*(-2*b
*Cos[x]*Sec[x/2]^4*Sin[x] + 4*a*Sec[x/2]^2*Tan[x/2] + 2*b*Cos[x]^2*Sec[x/2]
^4*Tan[x/2])*(-1 + Tan[x/2]^2))/(b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2)^(3
/2) + (Sqrt[a - b]*Sec[x/2]^2*Tan[x/2])/Sqrt[b*Cos[x]^2*Sec[x/2]^4 + 4*a*Ta
n[x/2]^2]))/(1 + ((a - b)*(-1 + Tan[x/2]^2)^2)/(b*Cos[x]^2*Sec[x/2]^4 + 4*a
*Tan[x/2]^2)) + ((a - 2*b)*((Sqrt[b]*Sec[x/2]^2*Tan[x/2])/Sqrt[b*Cos[x]^2*S
ec[x/2]^4 + 4*a*Tan[x/2]^2] - ((-2*b*Cos[x]*Sec[x/2]^4*Sin[x] + 4*a*Sec[x/2
]^2*Tan[x/2] + 2*b*Cos[x]^2*Sec[x/2]^4*Tan[x/2])*(2*a + b*(-1 + Tan[x/2]^2)
))/(2*Sqrt[b]*(b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2)^(3/2))))/(1 - (2*a +
 b*(-1 + Tan[x/2]^2))^2/(b*(b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2)))))/(Sq
rt[2]*Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]))
```

fricas [B] time = 0.49, size = 768, normalized size = 8.63

$$\left[\frac{2\sqrt{-a+b}\,b\log\left(-(a-b)\cos(2x)-\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)+b\right)\sin(2x)-(a-2b)\sqrt{b}\,\log\left(\frac{(a-2b)\,\cdots}{\cdots}\right.\cdots}{4b\sin(2x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a + b)*b*log(-(a - b)*cos(2*x) - sqrt(-a + b)*sqrt(((a - b)*c
os(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b)*sin(2*x) - (a - 2*b)*sqrt(b)
*log(((a - 2*b)*cos(2*x) - 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2
*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) - 2*(b*cos(2*x) + b)
*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x)), 1/4*(4*sqrt
(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1
))*sin(2*x)/((a - b)*cos(2*x) + a - b))*sin(2*x) - (a - 2*b)*sqrt(b)*log(((
a - 2*b)*cos(2*x) - 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1
))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) - 2*(b*cos(2*x) + b)*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x)), 1/2*((a - 2*b)*sqr
```

```
t(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2
*x)/(b*cos(2*x) + b))*sin(2*x) + sqrt(-a + b)*b*log(-(a - b)*cos(2*x) - sqr
t(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b)*sin
(2*x) - (b*cos(2*x) + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(
b*sin(2*x)), 1/2*(2*sqrt(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b))*sin(2*x) +
(a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x
) - 1))*sin(2*x)/(b*cos(2*x) + b))*sin(2*x) - (b*cos(2*x) + b)*sqrt(((a - b
)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(x))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Erro
r: Bad Argument Value
```

maple [B] time = 0.18, size = 174, normalized size = 1.96

$$-\frac{\cot(x)\sqrt{a+b\left(\cot^2(x)\right)}}{2}-\frac{a\ln\left(\cot(x)\sqrt{b}+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{2\sqrt{b}}+\sqrt{b}\,\ln\left(\cot(x)\sqrt{b}+\sqrt{a+b\left(\cot^2(x)\right)}\right)-\frac{\sqrt{b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -1/2*cot(x)*(a+b*cot(x)^2)^(1/2)-1/2*a/b^(1/2)*ln(cot(x)*b^(1/2)+(a+b*cot(x
)^2)^(1/2))+b^(1/2)*ln(cot(x)*b^(1/2)+(a+b*cot(x)^2)^(1/2))-(b^4*(a-b))^(1/
2)/b/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))+
a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x
)^2)^(1/2)*cot(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cot(x)^2+a}\,\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^2 + a)*cot(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x)^2\,\sqrt{b\cot(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(cot(x)^2*(a + b*cot(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cot^2(x)}\,\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*cot(x)**2, x)
```

3.23 $\int \sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=65

$$-\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

```
[Out] -arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-arctanh(cot(x)
*b^(1/2)/(a+b*cot(x)^2)^(1/2))*b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3661, 402, 217, 206, 377, 203}

$$-\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]) - Sqrt[b]*
ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
```

```
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\cot^2(x)}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)$$
$$= -\left(b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\right) + (-a+b)\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{cc}\right.$$
$$= -\left(b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)\right) + (-a+b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x,\right.$$
$$= -\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

Mathematica [C] time = 0.41, size = 167, normalized size = 2.57

$$\frac{1}{2}i\left(\sqrt{a-b}\,\log\left(-\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\cot^2(x)}+a-ib\cot(x)\right)}{(a-b)^{3/2}(\cot(x)+i)}\right) - \sqrt{a-b}\,\log\left(\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\cot^2(x)}+a+ib\,\text{c}\right.}{(a-b)^{3/2}(\cot(x)-i)}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] (I/2)*(Sqrt[a - b]*Log[((-4*I)*(a - I*b*Cot[x] + Sqrt[a - b]*Sqrt[a + b*Cot
[x]^2]))/((a - b)^(3/2)*(I + Cot[x]))] - Sqrt[a - b]*Log[((4*I)*(a + I*b*Co
t[x] + Sqrt[a - b]*Sqrt[a + b*Cot[x]^2]))/((a - b)^(3/2)*(-I + Cot[x]))] +
(2*I)*Sqrt[b]*Log[b*Cot[x] + Sqrt[b]*Sqrt[a + b*Cot[x]^2]])
```

fricas [B] time = 0.51, size = 515, normalized size = 7.92

$$\left[\frac{1}{2}\sqrt{-a+b}\,\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)+b\right)+\frac{1}{2}\sqrt{b}\,\log\left(\frac{(a-2b)\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a + b)*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*
x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b) + 1/2*sqrt(b)*log(((a - 2*b)*cos(
2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) -
 a - 2*b)/(cos(2*x) - 1)), -sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*c
os(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b)) + 1/2
*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b
)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1)), sqrt(-b)*arctan(sqrt
(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) +
 b)) + 1/2*sqrt(-a + b)*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*
cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b), -sqrt(a - b)*arctan(-sqrt(
a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*co
s(2*x) + a - b)) + sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)
/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b))]
```

giac [B] time = 2.64, size = 210, normalized size = 3.23

$$-\frac{1}{2}\left(\frac{2\sqrt{-a+b}\,b\arctan\left(\frac{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2+a-2b}{2\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}}+\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(-a + b)*b*arctan(1/2*((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2
+ b*cos(x)^2 + a))^2 + a - 2*b)/sqrt(a*b - b^2))/sqrt(a*b - b^2) + sqrt(-a
+ b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2))*sgn
(sin(x)) - 1/2*(2*sqrt(-a + b)*b*arctan(sqrt(-a + b)*sqrt(b)/sqrt(a*b - b^2
)) - sqrt(a*b - b^2)*sqrt(-a + b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b))*s
gn(sin(x))/sqrt(a*b - b^2)
```

maple [B] time = 0.23, size = 137, normalized size = 2.11

$$-\sqrt{b}\,\ln\left(\cot(x)\sqrt{b}+\sqrt{a+b\left(\cot^2(x)\right)}\right)+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{(a-b)b^2\cot(x)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(x)\right)}}\right)}{b(a-b)}-\frac{a\sqrt{b^4(a-b)}\,\arctan\left(\frac{}{\sqrt{}}\right.}{b^2(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -b^(1/2)*ln(cot(x)*b^(1/2)+(a+b*cot(x)^2)^(1/2))+(b^4*(a-b))^(1/2)/b/(a-b)*
arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))-a*(b^4*(a-b
))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*
cot(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\cot(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int((a + b*cot(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cot^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2), x)
```

3.24 $\int \sqrt{a+b\cot^2(x)}\,\tan^2(x)\,dx$

Optimal. Leaf size=51

$$\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\tan(x)\sqrt{a+b\cot^2(x)}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)+(a+b*cot(x)^2)^
(1/2)*tan(x)
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 475, 12, 377, 203}

$$\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\tan(x)\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2]*Tan[x]^2,x]
```

```
[Out] Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] + Sqrt[a + b*
Cot[x]^2]*Tan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cot^2(x)}\tan^2(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(1+x^2\right)}\,dx,x,\cot(x)\right)\\ &= \sqrt{a+b\cot^2(x)}\tan(x)-\text{Subst}\left(\int \frac{-a+b}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)\\ &= \sqrt{a+b\cot^2(x)}\tan(x)-(-a+b)\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)\\ &= \sqrt{a+b\cot^2(x)}\tan(x)-(-a+b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)\\ &= \sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\sqrt{a+b\cot^2(x)}\tan(x)\end{aligned}$$

Mathematica [C] time = 0.10, size = 44, normalized size = 0.86

$$\tan(x)\sqrt{a+b\cot^2(x)}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{(a-b)\cot^2(x)}{b\cot^2(x)+a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x]^2,x]
```

```
[Out] Sqrt[a + b*Cot[x]^2]*Hypergeometric2F1[-1/2, 1, 1/2, -(((a - b)*Cot[x]^2)/(
a + b*Cot[x]^2))]*Tan[x]
```

fricas [A] time = 0.58, size = 193, normalized size = 3.78

$$\left[\frac{1}{4}\sqrt{-a+b}\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2-4\left(a\tan(x)^3-(a-2b)\tan(x)\right)\sqrt{-a+b}}{\tan(x)^4+2\tan(x)^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8
*a*b + 8*b^2 - 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x
)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + sqrt((a*tan(x)^2 + b)/ta
n(x)^2)*tan(x), 1/2*sqrt(a - b)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/
tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + sqrt((a*tan(x)^2 + b)/tan(x)^2)*
tan(x)]
```

giac [B] time = 0.50, size = 239, normalized size = 4.69

$$\frac{1}{2}\left(\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)-\frac{4a\sqrt{-a+b}}{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(-a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2
+ a))^2) - 4*a*sqrt(-a + b)/((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*co
```

```
s(x)^2 + a))^2 - a))*sgn(sin(x)) - 1/2*(a*sqrt(-a + b)*log(-a - 2*sqrt(-a +
 b)*sqrt(b) + 2*b) - a*sqrt(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - sqr
t(-a + b)*b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + b^(3/2)*log(-a - 2*sqr
t(-a + b)*sqrt(b) + 2*b) + 2*a*sqrt(-a + b))*sgn(sin(x))/(a + sqrt(-a + b)*
sqrt(b) - b)
```

maple [B] time = 0.99, size = 752, normalized size = 14.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(1/2)*tan(x)^2,x)
```

```
[Out] 1/2*(-1+cos(x))*(cos(x)*b^(3/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos
(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))+cos(x)*(-a+b)^(1/2)*ln(-4*(-1+cos(
x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x
)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x
)^2/b^(1/2))*a-cos(x)*(-a+b)^(1/2)*ln(-4*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*c
os(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b
*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*b-cos(x)*(-a+
b)^(1/2)*ln(-2*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos
(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)
^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*a+cos(x)*(-a+b)^(1/2)*ln(-2*(-1+cos(x))
*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b
*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2
/b^(1/2))*b-cos(x)*b^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^
2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2
-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a-cos(x)*b^(1/2)*(-a+b)^(1/2)*(-(a*cos(
x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+
1)^2)^(1/2)*(-a+b)^(1/2)*b^(1/2))*((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))
^(1/2)/cos(x)/sin(x)/(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*4^(1/2
)/(-a+b)^(1/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cot(x)^2 + a} \tan(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^2 + a)*tan(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)^2 \sqrt{b \cot(x)^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2*(a + b*cot(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cot^2(x)} \tan^2(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*tan(x)**2, x)
```

3.25 $\int \sqrt{a+b\cot^2(x)}\,\tan^4(x)\,dx$

Optimal. Leaf size=85

$$-\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}-\frac{(3a-b)\tan(x)\sqrt{a+b\cot^2(x)}}{3a}$$

```
[Out] -arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-1/3*(3*a-b)*(a
+b*cot(x)^2)^(1/2)*tan(x)/a+1/3*(a+b*cot(x)^2)^(1/2)*tan(x)^3
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 475, 583, 12, 377, 203}

$$\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}-\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(3a-b)\tan(x)\sqrt{a+b\cot^2(x)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2]*Tan[x]^4,x]
```

```
[Out] -(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]) - ((3*a -
b)*Sqrt[a + b*Cot[x]^2]*Tan[x])/(3*a) + (Sqrt[a + b*Cot[x]^2]*Tan[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \sqrt{a+b\cot^2(x)}\,\tan^4(x)\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\left(1+x^2\right)}\,dx, x, \cot(x)\right)$$

$$= \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) - \frac{1}{3}\text{Subst}\left(\int \frac{-3a+b-2bx^2}{x^2\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)$$

$$= -\frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) + \frac{\text{Subst}\left(\int -\frac{3}{(1+x}\right.}{}$$

$$= -\frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) + (-a+b)\,\text{Subst}$$

$$= -\frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) + (-a+b)\,\text{Subst}$$

$$= -\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b}$$

Mathematica [C] time = 1.65, size = 174, normalized size = 2.05

$$\frac{1}{3}\sin^2(x)\tan^3(x)\sqrt{a+b\cot^2(x)}\left(\frac{b\cot^2(x)}{a}+1\right)\left(\frac{\csc^2(x)\left(a-2b\cot^2(x)\right)\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\,\sin^{-1}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\right.}{\left(a+b\cot^2(x)\right)\sqrt{\frac{b\cos^2(x)}{a}+\sin^2(x)}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x]^4,x]
```

```
[Out] (Sqrt[a + b*Cot[x]^2]*(1 + (b*Cot[x]^2)/a)*Sin[x]^2*((-4*(a - b)*Cos[x]^2*(
a + b*Cot[x]^2)*Hypergeometric2F1[2, 2, 3/2, ((a - b)*Cos[x]^2)/a])/a^2 + (
(a - 2*b*Cot[x]^2)*Csc[x]^2*(ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Sqrt[((a -
b)*Cos[x]^2)/a] + Sqrt[(b*Cos[x]^2)/a + Sin[x]^2]))/((a + b*Cot[x]^2)*Sqrt[
(b*Cos[x]^2)/a + Sin[x]^2]))*Tan[x]^3)/3
```

fricas [A] time = 0.59, size = 239, normalized size = 2.81

$$\left[\frac{3a\sqrt{-a+b}\,\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2+4\left(a\tan(x)^3-(a-2b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{\tan(x)^4+2\tan(x)^2+1}\right)+4\left(a\tan(x)^3\right.}{12a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a
^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a
*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + 4*(a*tan(x)^3 - (3
*a - b)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/a, -1/6*(3*sqrt(a - b)*a*a
rctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a
+ 2*b)) - 2*(a*tan(x)^3 - (3*a - b)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2)
)/a]
```

giac [B] time = 0.25, size = 476, normalized size = 5.60

$$-\frac{1}{6}\left(3\sqrt{-a+b}\,\log\left(\left(\sqrt{-a+b}\,\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)-\frac{4\left(3\left(\sqrt{-a+b}\,\cos(x)-\sqrt{-a\cos(x)^2+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*sqrt(-a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)
^2 + a))^2) - 4*(3*(sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a
))^4*(2*a - b)*sqrt(-a + b) - 6*(sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b
*cos(x)^2 + a))^2*a^2*sqrt(-a + b) + (4*a^3 - a^2*b)*sqrt(-a + b))/((sqrt(-
a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2 - a)^3)*sgn(sin(x)) +
 1/6*(3*a^2*sqrt(-a + b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 9*a^2*sqr
t(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 15*a*sqrt(-a + b)*b*log(-a -
2*sqrt(-a + b)*sqrt(b) + 2*b) + 21*a*b^(3/2)*log(-a - 2*sqrt(-a + b)*sqrt(b
) + 2*b) + 12*sqrt(-a + b)*b^2*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 12*
b^(5/2)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 8*a^2*sqrt(-a + b) - 18*a^
2*sqrt(b) - 24*a*sqrt(-a + b)*b + 30*a*b^(3/2) + 12*sqrt(-a + b)*b^2 - 12*b
^(5/2))*sgn(sin(x))/(a^2 + 3*a*sqrt(-a + b)*sqrt(b) - 5*a*b - 4*sqrt(-a + b
)*b^(3/2) + 4*b^2)
```

maple [B] time = 0.77, size = 951, normalized size = 11.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(1/2)*tan(x)^4,x)
```

```
[Out] -1/6*(-1+cos(x))*(cos(x)^3*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*
(-a+b)^(1/2)*b^(3/2)+3*cos(x)^3*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos
(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^(3/2)*a-4*cos(x)^3*(-(a*cos(x)^2
-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)*b^(1/2)*a+3*cos(x)^3*ln(-4*
(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)
+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+
a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a^2-3*cos(x)^3*ln(-4*(-1+cos(x))*(cos(x)*
b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(
-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))
*(-a+b)^(1/2)*a*b-3*cos(x)^3*ln(-2*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*cos(x)^
2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*cos(x
)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a^2+3*
cos(x)^3*ln(-2*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos
(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)
^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a*b-3*cos(x)^3*ln(4*cos(x)
*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*
```

```
b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^
(1/2)*a^2+cos(x)^2*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(
1/2)*b^(3/2)-4*cos(x)^2*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a
+b)^(1/2)*b^(1/2)*a+cos(x)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*
(-a+b)^(1/2)*b^(1/2)*a+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+
b)^(1/2)*a*b^(1/2))*((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))^(1/2)/cos(x)^
3/sin(x)/(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*4^(1/2)/(-a+b)^(1/
2)/a/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cot(x)^2 + a} \tan(x)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*cot(x)^2 + a)*tan(x)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x)^4 \sqrt{b \cot(x)^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4*(a + b*cot(x)^2)^(1/2),x)
[Out] int(tan(x)^4*(a + b*cot(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cot^2(x)} \tan^4(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2)*tan(x)**4,x)
[Out] Integral(sqrt(a + b*cot(x)**2)*tan(x)**4, x)
```

3.26 $\int \cot^3(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$-\frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b}+\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}+(a-b)\sqrt{a+b\cot^2(x)}-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))+1/3*(a+b*cot(x)^2)^(
3/2)-1/5*(a+b*cot(x)^2)^(5/2)/b+(a-b)*(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 80, 50, 63, 208}

$$-\frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b}+\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}+(a-b)\sqrt{a+b\cot^2(x)}-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) + (a - b)*Sqrt[a
 + b*Cot[x]^2] + (a + b*Cot[x]^2)^(3/2)/3 - (a + b*Cot[x]^2)^(5/2)/(5*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(x)\left(a+b\cot^2(x)\right)^{3/2}dx = -\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^{3/2}}{1+x^2}dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1+x}dx, x, \cot^2(x)\right)\right)$$
$$= -\frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}dx, x, \cot^2(x)\right)$$
$$= \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{1}{2}(a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}dx, x\right.$$
$$= (a-b)\sqrt{a+b\cot^2(x)} + \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{1}{2}(a-b)^2$$
$$= (a-b)\sqrt{a+b\cot^2(x)} + \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{(a-b)^2}{}$$
$$= -(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) + (a-b)\sqrt{a+b\cot^2(x)} + \frac{1}{3}\left(a+b\cot\right.$$

Mathematica [A] time = 0.50, size = 91, normalized size = 1.03

$$(a-b)^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)\right) - \frac{\sqrt{a+b\cot^2(x)}\left(3a^2+b(6a-5b)\cot^2(x)-20ab+3b^2\cot^4(x)+15b^2\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) - (Sqrt[a + b*Co
t[x]^2]*(3*a^2 - 20*a*b + 15*b^2 + (6*a - 5*b)*b*Cot[x]^2 + 3*b^2*Cot[x]^4)
)/(15*b)
```

fricas [B] time = 0.61, size = 486, normalized size = 5.52

$$\left[-\frac{15\left(\left(ab-b^2\right)\cos(2x)^2+ab-b^2-2\left(ab-b^2\right)\cos(2x)\right)\sqrt{a-b}\,\log\left(-2\left(a^2-2ab+b^2\right)\cos(2x)^2-2a^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*((a*b - b^2)*cos(2*x)^2 + a*b - b^2 - 2*(a*b - b^2)*cos(2*x))*sq
rt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 - 2*((a - b)*
cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a
 - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) + 4*((3*a^2 - 26*a*b + 23*b
^2)*cos(2*x)^2 + 3*a^2 - 14*a*b + 13*b^2 - 2*(3*a^2 - 20*a*b + 12*b^2)*cos(
2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x)^2 - 2*b*
cos(2*x) + b), -1/30*(15*((a*b - b^2)*cos(2*x)^2 + a*b - b^2 - 2*(a*b - b^2
)*cos(2*x))*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a -
b)/(cos(2*x) - 1))*(cos(2*x) - 1)/((a - b)*cos(2*x) - a)) + 2*((3*a^2 - 26*
a*b + 23*b^2)*cos(2*x)^2 + 3*a^2 - 14*a*b + 13*b^2 - 2*(3*a^2 - 20*a*b + 12
*b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x
)^2 - 2*b*cos(2*x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(x))]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%
%%{2,[0,2,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%
%%{-2,[1,4,1]%%%}+%%%{6,[1,2,1]%%%}+%%%{-4,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%%
%{-4,[0,2,2]%%%}+%%%{4,[0,0,2]%%%}] at parameters values [86,-97,-82]Warnin
g, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%
}+%%%{4,[0,2]%%%}] at parameters values [90.79236355,54.1277311612]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [69.8278764193,63.4443001123]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [108.020125429,82.1195442914]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [26.4357969165,7.79369851155]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [150.357303702,71.707969239]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[b+46,44]Unable to conver
t to real 44.0*(b+46.0)-1936.0 Error: Bad Argument ValueWarning, choosing r
oot of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]
%%%}] at parameters values [135.979061965,73.519035968]Warning, need to cho
ose a branch for the root of a polynomial with parameters. This might be wr
ong.The choice was done assuming [a,b]=[b+75,47]Unable to convert to real 4
7.0*(b+75.0)-2209.0 Error: Bad Argument ValueWarning, choosing root of [1,0
,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at pa
rameters values [141.604341501,50.5901726987]Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.The ch
oice was done assuming [a,b]=[b+15,55]Unable to convert to real 55.0*(b+15.
0)-3025.0 Error: Bad Argument ValueWarning, choosing root of [1,0,%%%{-2,[1
,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{2,[0,2,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[
2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{-2,[1,4,1]%%%}+%%%{6,[1,2,1]%%%}+%%%{-4,[1
,0,1]%%%}+%%%{1,[0,4,2]%%%}+%%%{-4,[0,2,2]%%%}+%%%{4,[0,0,2]%%%}] at parame
ters values [63,-64,2]Warning, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%
{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [113.23
8665889,81.3883557492]Warning, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%
```

```
{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [49.423
3726808,10.4309062702]Warning, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%
{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [90.780
3645204,82.7280518371]Warning, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%
{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [92.826
2473457,64.3995612673]Warning, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%
{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [106.15
9791361,66.1769613782]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[b+95,89]Unable to convert to real 89.0*(b+95.0)-7921.0 Error: Bad Ar
gument ValueWarning, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%
%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [101.17473746,17.
6881634681]Warning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming [a,b]=[b+53
,39]Unable to convert to real 39.0*(b+53.0)-1521.0 Error: Bad Argument Valu
eWarning, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[
1,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [96.452219774,89.629912049]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [a,b]=[b+46,66]Unable t
o convert to real 66.0*(b+46.0)-4356.0 Error: Bad Argument ValueUnable to c
ancel step at 0 of 2*(((15*a^2-60*a*b+45*b^2)*sqrt(a-b)*(sqrt(a*sin(x)^2-b*
sin(x)^2+b)-sqrt(a-b)*sin(x))^8+(90*a*b^2-90*b^3)*sqrt(a-b)*(sqrt(a*sin(x)^
2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^6+(30*a^2*b^2-170*a*b^3+140*b^4)*sqrt(a-b
)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^4+(70*a*b^4-70*b^5)*sqrt
(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2+(3*a^2*b^4-26*a*b^
5+23*b^6)*sqrt(a-b))/15/((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2
-b)^5+(a-b)*sqrt(a-b)/4*ln((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))
^2))--2*(((15*a^2-60*a*b+45*b^2)*sqrt(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-s
qrt(a-b)*sin(x))^8+(90*a*b^2-90*b^3)*sqrt(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+
b)-sqrt(a-b)*sin(x))^6+(30*a^2*b^2-170*a*b^3+140*b^4)*sqrt(a-b)*(sqrt(a*sin
(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^4+(70*a*b^4-70*b^5)*sqrt(a-b)*(sqrt(a
*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2+(3*a^2*b^4-26*a*b^5+23*b^6)*sqr
t(a-b))/15/((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2-b)^5+(a-b)*s
qrt(a-b)/4*ln((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2))Discontin
uities at zeroes of sin(x) were not checkedEvaluation time: 2.44Done
```

maple [B] time = 0.13, size = 150, normalized size = 1.70

$$-\frac{\left(a+b\left(\cot^2(x)\right)\right)^{\frac{5}{2}}}{5b}+\frac{b\left(\cot^2(x)\right)\sqrt{a+b\left(\cot^2(x)\right)}}{3}+\frac{4a\sqrt{a+b\left(\cot^2(x)\right)}}{3}-b\sqrt{a+b\left(\cot^2(x)\right)}+\frac{b^2\arctan\left(\frac{\sqrt{a+b}}{\sqrt{}}\right.}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] -1/5*(a+b*cot(x)^2)^(5/2)/b+1/3*b*cot(x)^2*(a+b*cot(x)^2)^(1/2)+4/3*a*(a+b*
cot(x)^2)^(1/2)-b*(a+b*cot(x)^2)^(1/2)+b^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^
2)^(1/2)/(-a+b)^(1/2))-2*a*b/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b
)^(1/2))+a^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 11.13, size = 120, normalized size = 1.36

$$\left(\frac{a}{3b}-\frac{a-b}{3b}\right)\left(b\cot(x)^2+a\right)^{3/2}-\frac{\left(b\cot(x)^2+a\right)^{5/2}}{5b}+(a-b)\left(\frac{a}{b}-\frac{a-b}{b}\right)\sqrt{b\cot(x)^2+a}+\mathrm{atan}\left(\frac{(a-b)^{3/2}\sqrt{b\cot}}{a^2-2ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atan(((a - b)^(3/2)*(a + b*cot(x)^2)^(1/2)*1i)/(a^2 - 2*a*b + b^2))*(a - b)
^(3/2)*1i - (a + b*cot(x)^2)^(5/2)/(5*b) + (a/(3*b) - (a - b)/(3*b))*(a + b
*cot(x)^2)^(3/2) + (a - b)*(a/b - (a - b)/b)*(a + b*cot(x)^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\cot^2(x)\right)^{\frac{3}{2}}\cot^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*cot(x)**3, x)
```

3.27 $\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{8\sqrt{b}}-\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)}-\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}+(a-b)^{\cdots}$$

```
[Out] (a-b)^(3/2)*arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))-1/8*(3*a^2-12*a
*b+8*b^2)*arctanh(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)-1/8*(5*a-4*b
)*cot(x)*(a+b*cot(x)^2)^(1/2)-1/4*b*cot(x)^3*(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3670, 477, 582, 523, 217, 206, 377, 203}

$$-\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{8\sqrt{b}}-\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}-\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)}+(a-b)^{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - ((3*a^2 -
 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]])/(8*Sqrt[b]
) - ((5*a - 4*b)*Cot[x]*Sqrt[a + b*Cot[x]^2])/8 - (b*Cot[x]^3*Sqrt[a + b*Co
t[x]^2])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
```

```
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \cot(x)\right)$$

$$= -\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(a(4a-3b)+(5a-4b)bx^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \ldots\right.$$

$$= -\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} + \frac{\text{Subst}\left(\int \frac{a(5a \ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} + (a-b)^2\,\text{Subs}\ldots$$

$$= -\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} + (a-b)^2\,\text{Subs}\ldots$$

$$= (a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{8\sqrt{b}} - \ldots$$

Mathematica [A] time = 1.19, size = 253, normalized size = 1.99

$$\frac{\csc(x)\sqrt{(a-b)\cos(2x)-a-b}\left(\sqrt{a-b}\left(\sqrt{-b}\cot(x)\csc(x)\sqrt{(a-b)\cos(2x)-a-b}\left(5a+2b\csc^2(x)-6b\right)-\right.\right.}{8\sqrt{2}\sqrt{-b}\sqrt{a-b}\sqrt{-\left(\csc^2(x)((a-\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[-a - b + (a - b)*Cos[2*x]]*Csc[x]*(8*Sqrt[2]*(a - b)^2*Sqrt[-b]*ArcTa
nh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos[2*x]]] + Sqrt[a -
 b]*(-(Sqrt[2]*(3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/S
qrt[-a - b + (a - b)*Cos[2*x]]]) + Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]]
*Cot[x]*Csc[x]*(5*a - 6*b + 2*b*Csc[x]^2))))/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-b
]*Sqrt[-((-a - b + (a - b)*Cos[2*x])*Csc[x]^2)])
```

fricas [B] time = 0.68, size = 1134, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)*log(-(a - b)*cos(2
*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)
 + b)*sin(2*x) - (3*a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*cos(2*x
))*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a -
 b)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) + 2*(4*b^2
*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2 + 5*a*b - 2*b^2)*sqrt(((a - b)*cos(2
*x) - a - b)/(cos(2*x) - 1)))/((b*cos(2*x) - b)*sin(2*x)), -1/8*((3*a^2 - 1
2*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*cos(2*x))*sqrt(-b)*arctan(sqrt(-b)
*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b))
*sin(2*x) - 4*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)*log(-(a - b)*
cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin
(2*x) + b)*sin(2*x) - (4*b^2*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2 + 5*a*b
- 2*b^2)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/((b*cos(2*x) - b)
*sin(2*x)), -1/16*(16*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(a - b)*arctan
(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a
- b)*cos(2*x) + a - b))*sin(2*x) + (3*a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*
b + 8*b^2)*cos(2*x))*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a -
 b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*s
in(2*x) - 2*(4*b^2*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2 + 5*a*b - 2*b^2)*s
qrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/((b*cos(2*x) - b)*sin(2*x))
, -1/8*(8*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(a - b)*arctan(-sqrt(a - b
)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x
) + a - b))*sin(2*x) + (3*a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*c
os(2*x))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x)
 - 1))*sin(2*x)/(b*cos(2*x) + b))*sin(2*x) - (4*b^2*cos(2*x) - (5*a*b - 6*b
^2)*cos(2*x)^2 + 5*a*b - 2*b^2)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) -
 1)))/((b*cos(2*x) - b)*sin(2*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(x))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Erro
r: Bad Argument Value
```

maple [B] time = 0.13, size = 286, normalized size = 2.25

$$-\frac{\cot(x)\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{3a\cot(x)\sqrt{a+b\left(\cot^2(x)\right)}}{8}-\frac{3a^2\ln\left(\cot(x)\sqrt{b}+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{8\sqrt{b}}+\frac{b\cot(x)\sqrt{a+b}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] -1/4*cot(x)*(a+b*cot(x)^2)^(3/2)-3/8*a*cot(x)*(a+b*cot(x)^2)^(1/2)-3/8*a^2/
b^(1/2)*ln(cot(x)*b^(1/2)+(a+b*cot(x)^2)^(1/2))+1/2*b*cot(x)*(a+b*cot(x)^2)
^(1/2)+3/2*b^(1/2)*a*ln(cot(x)*b^(1/2)+(a+b*cot(x)^2)^(1/2))-b^(3/2)*ln(cot
(x)*b^(1/2)+(a+b*cot(x)^2)^(1/2))+(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/
(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))-2*a/b*(b^4*(a-b))^(1/2)/(a-b
)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))+a^2*(b^4*
(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1
/2)*cot(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cot(x)^2+a\right)^{\frac{3}{2}}\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^2 + a)^(3/2)*cot(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot(x)^2\left(b\cot(x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2*(a + b*cot(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\cot^2(x)\right)^{\frac{3}{2}}\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*cot(x)**2, x)
```

3.28 $\int \cot(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$-(a-b)\sqrt{a+b\cot^2(x)}-\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}+(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

```
[Out] (a-b)^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))-1/3*(a+b*cot(x)^2)^(3
/2)-(a-b)*(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 50, 63, 208}

$$-(a-b)\sqrt{a+b\cot^2(x)}-\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}+(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - (a - b)*Sqrt[a +
b*Cot[x]^2] - (a + b*Cot[x]^2)^(3/2)/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx = -\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}\,dx,x,\cot^2(x)\right)\right)$$
$$= -\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{1}{2}(a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx,x,\cot^2(x)\right)$$
$$= -(a-b)\sqrt{a+b\cot^2(x)} - \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{1}{2}(a-b)^2\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+}}\right.$$
$$= -(a-b)\sqrt{a+b\cot^2(x)} - \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.}{b}$$
$$= (a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - (a-b)\sqrt{a+b\cot^2(x)} - \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/}$$

Mathematica [A] time = 0.17, size = 63, normalized size = 0.91

$$(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \frac{1}{3}\sqrt{a+b\cot^2(x)}\left(4a+b\cot^2(x)-3b\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - (Sqrt[a + b*Cot[x
]^2]*(4*a - 3*b + b*Cot[x]^2))/3
```

fricas [B] time = 0.67, size = 330, normalized size = 4.78

$$\left[-\frac{3\left((a-b)\cos(2x)-a+b\right)\sqrt{a-b}\,\log\left(-2\left(a^2-2ab+b^2\right)\cos(2x)^2-2a^2+b^2+2\left((a-b)\cos(2x)^2-(2a-\right.\right.}{12(\text{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a - b)*cos(2*x) - a + b)*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)
*cos(2*x)^2 - 2*a^2 + b^2 + 2*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)
*sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b
)*cos(2*x)) + 8*(2*(a - b)*cos(2*x) - 2*a + b)*sqrt(((a - b)*cos(2*x) - a -
 b)/(cos(2*x) - 1)))/(cos(2*x) - 1), 1/6*(3*((a - b)*cos(2*x) - a + b)*sqrt
(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1
))*(cos(2*x) - 1)/((a - b)*cos(2*x) - a)) - 4*(2*(a - b)*cos(2*x) - 2*a + b
)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(cos(2*x) - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(x))]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%
%%{2,[0,2,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%
%%{-2,[1,4,1]%%%}+%%%{6,[1,2,1]%%%}+%%%{-4,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%%
%{-4,[0,2,2]%%%}+%%%{4,[0,0,2]%%%}] at parameters values [86,-97,-82]Warnin
g, choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%
}+%%%{4,[0,2]%%%}] at parameters values [90.79236355,54.1277311612]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [69.8278764193,63.4443001123]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [108.020125429,82.1195442914]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [26.4357969165,7.79369851155]Warning,
 choosing root of [1,0,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+
%%%{4,[0,2]%%%}] at parameters values [150.357303702,71.707969239]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[b+63,75]Unable to conver
t to real 75.0*(b+63.0)-5625.0 Error: Bad Argument ValueWarning, choosing r
oot of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{2,[0,2,1]%%%},0,%%%{1
,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{-2,[1,4,1]%%%}+%%%{6,
[1,2,1]%%%}+%%%{-4,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%%%{-4,[0,2,2]%%%}+%%%{4,[
0,0,2]%%%}] at parameters values [18,-49,-33]Warning, choosing root of [1,0
,%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at pa
rameters values [70.2095400225,15.451549686]Warning, choosing root of [1,0,
%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at par
ameters values [100.356811349,81.9516051291]Warning, choosing root of [1,0,
%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at par
ameters values [133.032670634,51.6443148847]Warning, choosing root of [1,0,
%%%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at par
ameters values [42.28121641,31.8503101398]Warning, choosing root of [1,0,%%
%{-2,[1,0]%%%},0,%%%{1,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{4,[0,2]%%%}] at param
eters values [92.8262473457,64.3995612673]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a,b]=[b+66,40]Unable to convert to real 40.0*(b+66.0)-
1600.0 Error: Bad Argument ValueUnable to cancel step at 0 of 2*(((6*a*b-6*
b^2)*sqrt(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^4+(-6*a*b^2
+6*b^3)*sqrt(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2+(4*a*b
^3-4*b^4)*sqrt(a-b))/3/((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))^2-
b)^3+(-a+b)*sqrt(a-b)/4*ln((sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*sin(x))
^2))--2*(((6*a*b-6*b^2)*sqrt(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-b)*
sin(x))^4+(-6*a*b^2+6*b^3)*sqrt(a-b)*(sqrt(a*sin(x)^2-b*sin(x)^2+b)-sqrt(a-
b)*sin(x))^2+(4*a*b^3-4*b^4)*sqrt(a-b))/3/((sqrt(a*sin(x)^2-b*sin(x)^2+b)-s
qrt(a-b)*sin(x))^2-b)^3+(-a+b)*sqrt(a-b)/4*ln((sqrt(a*sin(x)^2-b*sin(x)^2+b
)-sqrt(a-b)*sin(x))^2))Discontinuities at zeroes of sin(x) were not checked
Evaluation time: 0.72Done
```

maple [B] time = 0.09, size = 136, normalized size = 1.97

$$-\frac{b\left(\cot^2(x)\right)\sqrt{a+b\left(\cot^2(x)\right)}}{3}-\frac{4a\sqrt{a+b\left(\cot^2(x)\right)}}{3}+b\sqrt{a+b\left(\cot^2(x)\right)}-\frac{b^2\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}+\frac{2ab\,\mathrm{arcta}}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] -1/3*b*cot(x)^2*(a+b*cot(x)^2)^(1/2)-4/3*a*(a+b*cot(x)^2)^(1/2)+b*(a+b*cot(
x)^2)^(1/2)-b^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))+2*a*
b/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))-a^2/(-a+b)^(1/2)*a
rctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [**B**] time = 3.54, size = 70, normalized size = 1.01

$$\operatorname{atanh}\left(\frac{(a-b)^{3/2}\sqrt{b\cot(x)^2+a}}{a^2-2\,a\,b+b^2}\right)(a-b)^{3/2}-\frac{\left(b\cot(x)^2+a\right)^{3/2}}{3}-(a-b)\,\sqrt{b\cot(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh(((a - b)^(3/2)*(a + b*cot(x)^2)^(1/2))/(a^2 - 2*a*b + b^2))*(a - b)^(
3/2) - (a + b*cot(x)^2)^(3/2)/3 - (a - b)*(a + b*cot(x)^2)^(1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\cot^2(x)\right)^{\frac{3}{2}}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*cot(x), x)
```

3.29 $\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx$

Optimal. Leaf size=75

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-b\sqrt{a+b\cot^2(x)}-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

```
[Out] a^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))-(a-b)^(3/2)*arctanh((a+b*cot(
x)^2)^(1/2)/(a-b)^(1/2))-b*(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3670, 446, 84, 156, 63, 208}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-b\sqrt{a+b\cot^2(x)}-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x]^2)^(3/2)*Tan[x],x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - (a - b)^(3/2)*ArcTanh[Sqrt[
a + b*Cot[x]^2]/Sqrt[a - b]] - b*Sqrt[a + b*Cot[x]^2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(1+x^2\right)}\,dx,x,\cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x(1+x)}\,dx,x,\cot^2(x)\right)\right)\\
&= -b\sqrt{a+b\cot^2(x)}-\frac{1}{2}\text{Subst}\left(\int \frac{a^2+(2a-b)bx}{x(1+x)\sqrt{a+bx}}\,dx,x,\cot^2(x)\right)\\
&= -b\sqrt{a+b\cot^2(x)}-\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\cot^2(x)\right)+\frac{1}{2}(a-b)^2\,\text{Subst}\left(\right.\\
&= -b\sqrt{a+b\cot^2(x)}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cot^2(x)}\right)}{b}+\frac{(a-b)^2\,\text{Subst}\left(\right.}{}\\
&= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-b\sqrt{a+b\,\text{co}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 1.00

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-b\sqrt{a+b\cot^2(x)}-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x]^2)^(3/2)*Tan[x],x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - (a - b)^(3/2)*ArcTanh[Sqrt[
a + b*Cot[x]^2]/Sqrt[a - b]] - b*Sqrt[a + b*Cot[x]^2]
```

fricas [A] time = 1.78, size = 565, normalized size = 7.53

$$\left[\frac{1}{2}a^{\frac{3}{2}}\log\left(2\,a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)-\frac{1}{4}(a-b)^{\frac{3}{2}}\log\left(-\frac{\left(8\,a^2-8\,ab+b^2\right)\tan(x)^4+2\left(4\,a\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/2*a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*t
an(x)^2 + b) - 1/4*(a - b)^(3/2)*log(-((8*a^2 - 8*a*b + b^2)*tan(x)^4 + 2*(
4*a*b - 3*b^2)*tan(x)^2 + b^2 + 4*((2*a - b)*tan(x)^4 + b*tan(x)^2)*sqrt(a
- b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) - b*sqrt
((a*tan(x)^2 + b)/tan(x)^2), -sqrt(-a)*a*arctan(sqrt(-a)*sqrt((a*tan(x)^2 +
 b)/tan(x)^2)*tan(x)^2/(a*tan(x)^2 + b)) - 1/4*(a - b)^(3/2)*log(-((8*a^2 -
 8*a*b + b^2)*tan(x)^4 + 2*(4*a*b - 3*b^2)*tan(x)^2 + b^2 + 4*((2*a - b)*ta
```

```
n(x)^4 + b*tan(x)^2)*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4
 + 2*tan(x)^2 + 1)) - b*sqrt((a*tan(x)^2 + b)/tan(x)^2), 1/2*(-a + b)^(3/2)
*arctan(-2*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2/((2*a - b)
*tan(x)^2 + b)) + 1/2*a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2
 + b)/tan(x)^2)*tan(x)^2 + b) - b*sqrt((a*tan(x)^2 + b)/tan(x)^2), -sqrt(-a
)*a*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2/(a*tan(x)^2 +
b)) + 1/2*(-a + b)^(3/2)*arctan(-2*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x
)^2)*tan(x)^2/((2*a - b)*tan(x)^2 + b)) - b*sqrt((a*tan(x)^2 + b)/tan(x)^2)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(x))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Erro
r: Bad Argument Value
```

maple [C] time = 0.76, size = 2628, normalized size = 35.04

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(3/2)*tan(x),x)
```

```
[Out] sin(x)^2*((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))^(3/2)*(2*2^(1/2)*((cos(x
)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b
)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(
x)-a)/(cos(x)+1)/b)^(1/2)*EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a
+b)/b)^(1/2)/sin(x),((8*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8
*a*b+b^2)/b^2)^(1/2))*a*b*sin(x)*cos(x)-2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2
)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*
a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^
(1/2)*EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),
((8*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2)
)*b^2*sin(x)*cos(x)+2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1
/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2
)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((
-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-
b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b
)^(1/2)-2*a+b)/b)^(1/2))*a^2*sin(x)*cos(x)-4*2^(1/2)*((cos(x)*a^(1/2)*(a-b)
^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(co
s(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1
)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/s
in(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)
^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a*b*sin(x)*cos(x)+2*2^(1/2)
*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos
(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x
)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^
(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2
)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*b^2*
sin(x)*cos(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*
cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1
/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos
```

```
(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2
)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)
-2*a+b)/b)^(1/2))*a^2*sin(x)*cos(x)+2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-
a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^
(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1
/2)*EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),((
8*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))*
a*b*sin(x)-2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x
)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(
a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticF((-1+cos(x))*(
(2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),((8*a^(3/2)*(a-b)^(1/2)-4*a^(
1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))*b^2*sin(x)+2*2^(1/2)*((cos(
x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/
b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos
(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2
*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)
^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a^2*sin(x)-
4*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x
)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2
)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/
2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-
(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1
/2))*a*b*sin(x)+2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-
a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^
(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+c
os(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(
1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1
/2)-2*a+b)/b)^(1/2))*b^2*sin(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1
/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2
)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*
EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*
a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a
^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a^2*sin(x)+cos(x)^2*((2*a^(1/2)*(a-b)^(
1/2)-2*a+b)/b)^(1/2)*a*b-cos(x)^2*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)*b
^2-((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)*a*b)/(a*cos(x)^2-b*cos(x)^2-a)^2
/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cot(x)^2 + a\right)^{\frac{3}{2}} \tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^2 + a)^(3/2)*tan(x), x)
```

mupad [B] time = 0.54, size = 506, normalized size = 6.75

$$\operatorname{atanh}\left(\frac{2\,b^6\,\sqrt{a^3}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6} - \frac{8\,a\,b^5\,\sqrt{a^3}\,\sqrt{a+\frac{b}{\tan(x)^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6} + \frac{12\,a^2\,b^4\,\sqrt{a^3}\,\sqrt{a+ \cdots}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3 \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh((2*b^6*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 1
2*a^4*b^4 - 6*a^5*b^3) - (8*a*b^5*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^
```

```
2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) + (12*a^2*b^4*(a^3)^(1/2)*(a +
b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (6*a^
3*b^3*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b
^4 - 6*a^5*b^3))*(a^3)^(1/2) - atanh((2*a*b^5*(a + b/tan(x)^2)^(1/2)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^
4*b^4 + 6*a^5*b^3) - (6*a^2*b^4*(a + b/tan(x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b +
 a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b^4 + 6*a^5*
b^3) + (6*a^3*b^3*(a + b/tan(x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1
/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b^4 + 6*a^5*b^3))*((a - b)
^3)^(1/2) - b*(a + b/tan(x)^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^2(x)\right)^{\frac{3}{2}} \tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(3/2)*tan(x),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*tan(x), x)
```

3.30 $\int \left(a+b\cot^2(x)\right)^{3/2}\tan^2(x)\,dx$

Optimal. Leaf size=80

$$-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+a\tan(x)\sqrt{a+b\cot^2(x)}$$

```
[Out] (a-b)^(3/2)*arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))-b^(3/2)*arctanh
(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))+a*(a+b*cot(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 474, 523, 217, 206, 377, 203}

$$-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+a\tan(x)\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x]^2)^(3/2)*Tan[x]^2,x]
```

```
[Out] (a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - b^(3/2)*A
rcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] + a*Sqrt[a + b*Cot[x]^2]*Tan[
x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \left(a+b\cot^2(x)\right)^{3/2}\tan^2(x)\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\left(1+x^2\right)}\,dx,x,\cot(x)\right)$$

$$= a\sqrt{a+b\cot^2(x)}\,\tan(x) - \text{Subst}\left(\int \frac{-a(a-2b)+b^2x^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)$$

$$= a\sqrt{a+b\cot^2(x)}\,\tan(x) + (a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)$$

$$= a\sqrt{a+b\cot^2(x)}\,\tan(x) + (a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

$$= (a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) + a\sqrt{a+b\cot^2(x)}\,\tan(x)$$

Mathematica [B] time = 0.74, size = 222, normalized size = 2.78

$$\frac{\sin(x)\sqrt{-\left(\csc^2(x)((a-b)\cos(2x)-a-b)\right)}\left(\sqrt{a-b}\left(\sqrt{2}\,b^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{-b}\cos(x)}{\sqrt{(a-b)\cos(2x)-a-b}}\right)+a\sqrt{-b}\sec(x)\sqrt{(a-b)\cos(2x)-a-b}\right)-\dots\right)}{\sqrt{2}\sqrt{-b}\sqrt{a-b}\sqrt{(a-b)\cos(2x)-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x]^2)^(3/2)*Tan[x]^2,x]
```

```
[Out] (Sqrt[-((-a - b + (a - b)*Cos[2*x])*Csc[x]^2)]*(-(Sqrt[2]*(a - b)^2*Sqrt[-b
]*ArcTanh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos[2*x]]]) +
Sqrt[a - b]*(Sqrt[2]*b^2*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/Sqrt[-a - b + (a
 - b)*Cos[2*x]]] + a*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]]*Sec[x]))*Sin[
x])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]])
```

fricas [A] time = 1.43, size = 543, normalized size = 6.79

$$\left[\frac{1}{4}(-a+b)^{\frac{3}{2}}\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2+4\left(a\tan(x)^3-(a-2b)\tan(x)\right)\sqrt{-\dots}}{\tan(x)^4+2\tan(x)^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(-a + b)^(3/2)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 -
 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan
(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*b^(3/2)*log((a*tan
(x)^2 - 2*sqrt(b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x) + 2*b)/tan(x)^2) +
 a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x), sqrt(-b)*b*arctan(sqrt(-b)*sqrt(
(a*tan(x)^2 + b)/tan(x)^2)*tan(x)/b) + 1/4*(-a + b)^(3/2)*log(-(a^2*tan(x)^
4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a -
 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*t
an(x)^2 + 1)) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x), 1/2*(a - b)^(3/2)
*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 -
a + 2*b)) + 1/2*b^(3/2)*log((a*tan(x)^2 - 2*sqrt(b)*sqrt((a*tan(x)^2 + b)/t
an(x)^2)*tan(x) + 2*b)/tan(x)^2) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)
, 1/2*(a - b)^(3/2)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*ta
n(x)/(a*tan(x)^2 - a + 2*b)) + sqrt(-b)*b*arctan(sqrt(-b)*sqrt((a*tan(x)^2
+ b)/tan(x)^2)*tan(x)/b) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.69, size = 1276, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(3/2)*tan(x)^2,x)
```

```
[Out] -1/2*((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))^(3/2)*(-1+cos(x))^3*(2*cos(x
)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-
4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)
^2)^(1/2))*b^(9/2)-4*cos(x)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)
^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^
2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^(7/2)*a+2*cos(x)*(-a+b)^(1/2)*b^(5/2
)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a+2*cos(x)*ln(4*cos(x)*(-
a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*c
os(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^(5/
2)*a^2+2*a*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(5/2)*(-a+b)^(
1/2)-cos(x)*(-a+b)^(1/2)*ln(-4*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/
(cos(x)+1)^2)^(1/2)-a*cos(x)+b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1
)^2)^(1/2)*b^(1/2)+a)/(-1+cos(x)))*b^4-3*cos(x)*(-a+b)^(1/2)*ln(-2*(-1+cos(
x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x
)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x
)^2/b^(1/2))*a^3*b+6*cos(x)*(-a+b)^(1/2)*ln(-2*(-1+cos(x))*(cos(x)*b^(1/2)*
(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(
x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*a^2*b^2
-3*cos(x)*(-a+b)^(1/2)*ln(-2*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)
/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*a*b^3+cos(x)*(-a+b)^(1/2)
*ln(-2*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2
)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b
^(1/2)+a)/sin(x)^2/b^(1/2))*b^4+3*cos(x)*(-a+b)^(1/2)*ln(-4*(-1+cos(x))*(co
s(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos
(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(
1/2))*a^3*b-6*cos(x)*(-a+b)^(1/2)*ln(-4*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*co
```

```
s(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*
cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*a^2*b^2+3*cos(
x)*(-a+b)^(1/2)*ln(-4*(-1+cos(x))*(cos(x)*b^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-
a)/(cos(x)+1)^2)^(1/2)+a*cos(x)-b*cos(x)+(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x
)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*a*b^3)/cos(x)/sin(x)^3/(-(a*cos(
x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(3/2)/b^(5/2)/(-a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cot(x)^2 + a\right)^{\frac{3}{2}} \tan(x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^2 + a)^(3/2)*tan(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x)^2 \left(b\cot(x)^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(tan(x)^2*(a + b*cot(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^2(x)\right)^{\frac{3}{2}} \tan^2(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(3/2)*tan(x)**2,x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*tan(x)**2, x)
```

3.31 $\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{8d}-\frac{b\cot(c+dx)\left(a+b\cot^2(c+dx)\right)^{3/2}}{4d}-\frac{b(7a-4b)\cot(c+dx)\sqrt{}}{8d}$$

```
[Out] -(a-b)^(5/2)*arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d-1/4*
b*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(3/2)/d-1/8*(15*a^2-20*a*b+8*b^2)*arctanh(c
ot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d-1/8*(7*a-4*b)*b*cot(d
*x+c)*(a+b*cot(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3661, 416, 528, 523, 217, 206, 377, 203}

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{8d}-\frac{b\cot(c+dx)\left(a+b\cot^2(c+dx)\right)^{3/2}}{4d}-\frac{b(7a-4b)\cot(c+dx)\sqrt{}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(5/2),x]
```

```
[Out] -(((a - b)^(5/2)*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^
2]])/d) - (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Cot[c + d*x])
/Sqrt[a + b*Cot[c + d*x]^2]])/(8*d) - ((7*a - 4*b)*b*Cot[c + d*x]*Sqrt[a +
b*Cot[c + d*x]^2])/(8*d) - (b*Cot[c + d*x]*(a + b*Cot[c + d*x]^2)^(3/2))/(4
*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{b\cot(c+dx)\left(a+b\cot^2(c+dx)\right)^{3/2}}{4d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\left(a(4a-b)+(7a-4b)bx^2\right)}{1+x^2}\,dx, x\right.}{4d}$$

$$= -\frac{(7a-4b)b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d} - \frac{b\cot(c+dx)\left(a+b\cot^2(c+dx\right.}{4d}$$

$$= -\frac{(7a-4b)b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d} - \frac{b\cot(c+dx)\left(a+b\cot^2(c+dx\right.}{4d}$$

$$= -\frac{(7a-4b)b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d} - \frac{b\cot(c+dx)\left(a+b\cot^2(c+dx\right.}{4d}$$

$$= -\frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\cot(c\right.}{\sqrt{a+b\cot^2(}}}{8d}$$

Mathematica [C] time = 1.69, size = 259, normalized size = 1.51

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\log\left(\sqrt{b}\sqrt{a+b\cot^2(c+dx)}+b\cot(c+dx)\right)+b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}\left(9a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(5/2),x]
```

```
[Out] -1/8*(b*Cot[c + d*x]*Sqrt[a + b*Cot[c + d*x]^2]*(9*a - 4*b + 2*b*Cot[c + d*
x]^2) - (4*I)*(a - b)^(5/2)*Log[((-4*I)*(a - I*b*Cot[c + d*x] + Sqrt[a - b]
*Sqrt[a + b*Cot[c + d*x]^2]))/((a - b)^(7/2)*(I + Cot[c + d*x]))] + (4*I)*(
a - b)^(5/2)*Log[((4*I)*(a + I*b*Cot[c + d*x] + Sqrt[a - b]*Sqrt[a + b*Cot[
c + d*x]^2]))/((a - b)^(7/2)*(-I + Cot[c + d*x]))] + Sqrt[b]*(15*a^2 - 20*a
*b + 8*b^2)*Log[b*Cot[c + d*x] + Sqrt[b]*Sqrt[a + b*Cot[c + d*x]^2]])/d
```

fricas [B] time = 0.53, size = 1520, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(-
a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x
 + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)*sin(2*d*x +
2*c) + (15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2)*cos(2*d*x + 2*c
))*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a - b)*cos(2*
d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*b)/(co
s(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - 2*(4*b^2*cos(2*d*x + 2*c) - 3*(3*a*
b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*sqrt(((a - b)*cos(2*d*x + 2*
c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x +
2*c)), 1/16*(16*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c))*
sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(co
s(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b))*s
in(2*d*x + 2*c) - (15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2)*cos(
2*d*x + 2*c))*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a
- b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a
 - 2*b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + 2*(4*b^2*cos(2*d*x + 2*c
) - 3*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*sqrt(((a - b)*cos
(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2*d*x + 2*c) - d)*s
in(2*d*x + 2*c)), -1/8*((15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2
)*cos(2*d*x + 2*c))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c)
 - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b)
)*sin(2*d*x + 2*c) + 4*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x +
 2*c))*sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a -
 b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)
*sin(2*d*x + 2*c) - (4*b^2*cos(2*d*x + 2*c) - 3*(3*a*b - 2*b^2)*cos(2*d*x +
 2*c)^2 + 9*a*b - 2*b^2)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x
 + 2*c) - 1)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c)), 1/8*(8*(a^2 - 2
*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(a - b)*arctan(-sqrt
(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin
(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b))*sin(2*d*x + 2*c) - (15*a^
2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2)*cos(2*d*x + 2*c))*sqrt(-b)*a
rctan(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) -
1))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b))*sin(2*d*x + 2*c) + (4*b^2*co
s(2*d*x + 2*c) - 3*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*sqrt
```

```
(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2*d*x
+ 2*c) - d)*sin(2*d*x + 2*c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(d*x+c))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Evaluation time: 2Error: Bad Argument Type
```

maple [B] time = 0.46, size = 462, normalized size = 2.70

$$-\frac{b^2\left(\cot^3(dx+c)\right)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{4d}-\frac{9ba\cot(dx+c)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{8d}-\frac{15\sqrt{b}\,a^2\ln\left(\cot(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^(5/2),x)
```

```
[Out] -1/4/d*b^2*cot(d*x+c)^3*(a+b*cot(d*x+c)^2)^(1/2)-9/8/d*b*a*cot(d*x+c)*(a+b*
cot(d*x+c)^2)^(1/2)-15/8/d*b^(1/2)*a^2*ln(cot(d*x+c)*b^(1/2)+(a+b*cot(d*x+c
)^2)^(1/2))+1/2/d*b^2*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(1/2)+5/2/d*b^(3/2)*a*l
n(cot(d*x+c)*b^(1/2)+(a+b*cot(d*x+c)^2)^(1/2))-1/d*b^(5/2)*ln(cot(d*x+c)*b^
(1/2)+(a+b*cot(d*x+c)^2)^(1/2))+1/d*b*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*
b^2/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))-3/d*a*(b^4*(a-b)
)^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*c
ot(d*x+c))+3/d*a^2/b*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(
1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))-1/d*a^3*(b^4*(a-b))^(1/2)/b^2/(a-
b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cot(dx+c)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\cot(c+d\,x)^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b*cot(c + d*x)^2)^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^2\left(c + dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**(5/2),x)
[Out] Integral((a + b*cot(c + d*x)**2)**(5/2), x)
```

3.32 $\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{2d}$$

```
[Out] -(a-b)^(3/2)*arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d-1/2*
(3*a-2*b)*arctanh(cot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d-1/
2*b*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3661, 416, 523, 217, 206, 377, 203}

$$-\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^
2]])/d) - ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[c + d*x])/Sqrt[a + b*Co
t[c + d*x]^2]])/(2*d) - (b*Cot[c + d*x]*Sqrt[a + b*Cot[c + d*x]^2])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx=-\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\cot(c+dx)\right)}{d}$$

$$=-\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}-\frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(c+dx)\right)}{2d}$$

$$=-\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}-\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(c+ \ldots$$

$$=-\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}-\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\cot(c+dx)}{\sqrt{a+b\cot^2(c \ldots}}$$

$$=-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{2d}-\frac{b\cot(\ldots}{}$$

Mathematica [C] time = 1.38, size = 234, normalized size = 1.86

$$\frac{-b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}+i(a-b)^{3/2}\log\left(-\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\cot^2(c+dx)}+a-ib\cot(c+dx)\right)}{(a-b)^{5/2}(\cot(c+dx)+i)}\right)-i(a-b)^{3/2}\log\left(\frac{4i\left(\sqrt{a-\ldots}\right.}{}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(3/2),x]
```

```
[Out] (-(b*Cot[c + d*x]*Sqrt[a + b*Cot[c + d*x]^2]) + I*(a - b)^(3/2)*Log[((-4*I)
*(a - I*b*Cot[c + d*x] + Sqrt[a - b]*Sqrt[a + b*Cot[c + d*x]^2]))/((a - b)^
(5/2)*(I + Cot[c + d*x]))] - I*(a - b)^(3/2)*Log[((4*I)*(a + I*b*Cot[c + d*
x] + Sqrt[a - b]*Sqrt[a + b*Cot[c + d*x]^2]))/((a - b)^(5/2)*(-I + Cot[c +
d*x]))] + Sqrt[b]*(-3*a + 2*b)*Log[b*Cot[c + d*x] + Sqrt[b]*Sqrt[a + b*Cot[
c + d*x]^2]])/(2*d)
```

fricas [B] time = 0.49, size = 1071, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)*sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) - sqrt(-a + b)*
sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x +
 2*c) + b)*sin(2*d*x + 2*c) + (3*a - 2*b)*sqrt(b)*log(((a - 2*b)*cos(2*d*x
+ 2*c) - 2*sqrt(b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c
) - 1))*sin(2*d*x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c
) + 2*(b*cos(2*d*x + 2*c) + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos
(2*d*x + 2*c) - 1)))/(d*sin(2*d*x + 2*c)), 1/2*((3*a - 2*b)*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*s
in(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b))*sin(2*d*x + 2*c) - (a - b)*sqrt(-
a + b)*log(-(a - b)*cos(2*d*x + 2*c) - sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x
 + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)*sin(2*d*x +
2*c) - (b*cos(2*d*x + 2*c) + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(co
s(2*d*x + 2*c) - 1)))/(d*sin(2*d*x + 2*c)), -1/4*(4*(a - b)^(3/2)*arctan(-s
qrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*
sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b))*sin(2*d*x + 2*c) + (3*
a - 2*b)*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) - 2*sqrt(b)*sqrt(((a - b)*
cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*
b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + 2*(b*cos(2*d*x + 2*c) + b)*sq
rt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/(d*sin(2*d*x
 + 2*c)), -1/2*(2*(a - b)^(3/2)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x
 + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*
x + 2*c) + a - b))*sin(2*d*x + 2*c) - (3*a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*
sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x +
 2*c)/(b*cos(2*d*x + 2*c) + b))*sin(2*d*x + 2*c) + (b*cos(2*d*x + 2*c) + b)
*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/(d*sin(2*
d*x + 2*c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(d*x+c))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Evaluation time: 0.71Error: Bad Argument Type
```

maple [B] time = 0.38, size = 298, normalized size = 2.37

$$-\frac{b\cot(dx+c)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{2d}-\frac{3\sqrt{b}\,a\ln\left(\cot(dx+c)\sqrt{b}+\sqrt{a+b\left(\cot^2(dx+c)\right)}\right)}{2d}+\frac{b^{\frac{3}{2}}\ln\left(\cot(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2*b*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(1/2)/d-3/2/d*b^(1/2)*a*ln(cot(d*x+c)*
b^(1/2)+(a+b*cot(d*x+c)^2)^(1/2))+1/d*b^(3/2)*ln(cot(d*x+c)*b^(1/2)+(a+b*co
```

```
t(d*x+c)^2)^(1/2))-1/d*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))
^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))+2/d*a/b*(b^4*(a-b))^(1/2)/(a-b)
*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))-1/
d*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*c
ot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cot\left(dx+c\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{cot}\left(c+d\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^(3/2),x)
[Out] int((a + b*cot(c + d*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\cot^{2}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**(3/2),x)
[Out] Integral((a + b*cot(c + d*x)**2)**(3/2), x)
```

3.33 $\int \sqrt{a+b\cot^2(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*(a-b)^(1/2)/d-arct
anh(cot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 402, 217, 206, 377, 203}

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[c + d*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]
])/d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]
)/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \cot(c+\right.}{d}$$

$$= -\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cot(c+a}{\sqrt{a+b\cot^2(}}\right.}{d}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

Mathematica [C] time = 0.61, size = 202, normalized size = 2.32

$$\frac{i\left(\sqrt{a-b}\,\log\left(-\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\cot^2(c+dx)}+a-ib\cot(c+dx)\right)}{(a-b)^{3/2}(\cot(c+dx)+i)}\right)-\sqrt{a-b}\,\log\left(\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\cot^2(c+dx)}+a+ib\cot(c+dx)\right)}{(a-b)^{3/2}(\cot(c+dx)-i)}\right)+2i\sqrt{b}\,\log\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[c + d*x]^2],x]
```

```
[Out] ((I/2)*(Sqrt[a - b]*Log[((-4*I)*(a - I*b*Cot[c + d*x] + Sqrt[a - b]*Sqrt[a
+ b*Cot[c + d*x]^2]))/((a - b)^(3/2)*(I + Cot[c + d*x]))] - Sqrt[a - b]*Log
[((4*I)*(a + I*b*Cot[c + d*x] + Sqrt[a - b]*Sqrt[a + b*Cot[c + d*x]^2]))/((
a - b)^(3/2)*(-I + Cot[c + d*x]))] + (2*I)*Sqrt[b]*Log[b*Cot[c + d*x] + Sqr
t[b]*Sqrt[a + b*Cot[c + d*x]^2]]))/d
```

fricas [B] time = 0.94, size = 703, normalized size = 8.08

$$\left[\frac{\sqrt{-a+b}\,\log\left(-(a-b)\cos(2\,dx+2\,c)+\sqrt{-a+b}\,\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\,\sin(2\,dx+2\,c)+b\right)+\sqrt{b}\,\log\left(\frac{(a-2\,b)\cos}{}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a -
b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)
+ sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a - b)*cos(2*d
*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*b)/(cos
```

```
(2*d*x + 2*c) - 1)))/d, -1/2*(2*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a -
b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a -
 b)*cos(2*d*x + 2*c) + a - b)) - sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) +
2*sqrt(b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*s
in(2*d*x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1)))/d, 1/2*(2*sqrt(-b)*arct
an(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))
*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b)) + sqrt(-a + b)*log(-(a - b)*cos
(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2
*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b))/d, -(sqrt(a - b)*arctan(-sqrt(a -
b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*
x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b)) - sqrt(-b)*arctan(sqrt(-b)*sqr
t(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*
c)/(b*cos(2*d*x + 2*c) + b)))/d]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(d*x+c))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Warning, replacing 0 by ` u`, a substitution variabl
e should perhaps be purged.Warning, replacing 0 by ` u`, a substitution var
iable should perhaps be purged.Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, integra
tion of abs or sign assumes constant sign by intervals (correct if the argu
ment is real):Check [abs(t_nostep)]Warning, need to choose a branch for the
 root of a polynomial with parameters. This might be wrong.Non regular valu
e [0] was discarded and replaced randomly by 0=[-92]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.Non regular value [0] was discarded and replaced randomly by 0=[4]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.Non regular value [0] was discarded and replaced randomly b
y 0=[69]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.Non regular value [0] was discarded and rep
laced randomly by 0=[-99]Warning, need to choose a branch for the root of a
 polynomial with parameters. This might be wrong.Non regular value [0] was
discarded and replaced randomly by 0=[94]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.Non regula
r value [0] was discarded and replaced randomly by 0=[-94]Warning, need to
choose a branch for the root of a polynomial with parameters. This might be
 wrong.Non regular value [0] was discarded and replaced randomly by 0=[-41]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.Non regular value [0] was discarded and replaced ra
ndomly by 0=[-81]Precision problem choosing root in common_EXT, current pre
```

```
cision 14Evaluation time: 0.63index.cc index_m operator + Error: Bad Argume
nt Value
```

maple [B] time = 0.42, size = 170, normalized size = 1.95

$$-\frac{\sqrt{b}\,\ln\left(\cot(dx+c)\sqrt{b}+\sqrt{a+b\left(\cot^2(dx+c)\right)}\right)}{d}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{(a-b)b^2\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{db(a-b)}-\frac{a\sqrt{b^4(a-b)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/d*b^(1/2)*ln(cot(d*x+c)*b^(1/2)+(a+b*cot(d*x+c)^2)^(1/2))+1/d*(b^4*(a-b)
)^(1/2)/b/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)
*cot(d*x+c))-1/d*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))
^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\cot(c+d\,x)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^(1/2),x)
```

```
[Out] int((a + b*cot(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cot^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(c + d*x)**2), x)
```

3.34 $\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d\sqrt{a-b}}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d/(a-b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3661, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cot[c + d*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/(Sqrt[a - b
]*d))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{\sqrt{a-b}\,d}$$

Mathematica [B] time = 0.41, size = 111, normalized size = 2.36

$$-\frac{\cot(c+dx)\sqrt{\frac{b\cot^2(c+dx)}{a}+1}\tanh^{-1}\left(\frac{\sqrt{-\frac{(a-b)\cot^2(c+dx)}{a}}}{\sqrt{\frac{b\cot^2(c+dx)}{a}+1}}\right)}{d\sqrt{-\frac{(a-b)\cot^2(c+dx)}{a}}\sqrt{a+b\cot^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cot[c + d*x]^2],x]
```

```
[Out] -((ArcTanh[Sqrt[-(((a - b)*Cot[c + d*x]^2)/a)]/Sqrt[1 + (b*Cot[c + d*x]^2)/
a]]*Cot[c + d*x]*Sqrt[1 + (b*Cot[c + d*x]^2)/a])/(d*Sqrt[-(((a - b)*Cot[c +
 d*x]^2)/a)]*Sqrt[a + b*Cot[c + d*x]^2]))
```

fricas [B] time = 0.48, size = 239, normalized size = 5.09

$$\left[-\frac{\sqrt{-a+b}\log\left(-2\left(a^2-2ab+b^2\right)\cos(2dx+2c)^2-2\left((a-b)\cos(2dx+2c)-b\right)\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2dx+2c)-a}{\cos(2dx+2c)-1}}\right.}{4(a-b)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 - 2*((a -
b)*cos(2*d*x + 2*c) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a -
b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*c
os(2*d*x + 2*c))/((a - b)*d), -1/2*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*
d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2
*d*x + 2*c) - b))/(sqrt(a - b)*d)]
```

giac [B] time = 4.94, size = 88, normalized size = 1.87

$$\frac{2\arctan\left(-\frac{\sqrt{b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+b}+\sqrt{b}}{2\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c)
^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(b)
)/sqrt(a - b))/(sqrt(a - b)*d)
```

maple [A] time = 0.39, size = 68, normalized size = 1.45

$$-\frac{\sqrt{b^4 (a-b)}\, \arctan\left(\frac{(a-b)b^2 \cot(dx+c)}{\sqrt{b^4(a-b)}\, \sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{d\, b^2 (a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/d*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*co
t(d*x+c)^2)^(1/2)*cot(d*x+c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 0.85, size = 41, normalized size = 0.87

$$-\frac{\mathrm{atan}\left(\frac{\cot(c+d\,x)\,\sqrt{a-b}}{\sqrt{b\,{\cot(c+d\,x)}^2+a}}\right)}{d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(1/2),x)
```

```
[Out] -atan((cot(c + d*x)*(a - b)^(1/2))/(a + b*cot(c + d*x)^2)^(1/2))/(d*(a - b)
^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cot^2(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cot(c + d*x)**2), x)
```

3.35 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{b\cot(c+dx)}{ad(a-b)\sqrt{a+b\cot^2(c+dx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d(a-b)^{3/2}}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(3/2)/d+b*co
t(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3661, 382, 377, 203}

$$\frac{b\cot(c+dx)}{ad(a-b)\sqrt{a+b\cot^2(c+dx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/((a - b)^(3
/2)*d)) + (b*Cot[c + d*x])/(a*(a - b)*d*Sqrt[a + b*Cot[c + d*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(c+dx)\right)}{(a-b)d}$$

$$= \frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{3/2}d} + \frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}}$$

Mathematica [C] time = 3.64, size = 231, normalized size = 2.72

$$-\frac{\cos^2(c+dx)\cot(c+dx)\left(4(a-b)^2\cos^2(c+dx)\left(a\tan^2(c+dx)+b\right)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\cos^2(c+dx)}{a}\right)-\frac{15a\left(3a\tan^2(c\right.}{\cdots}\right.}{15a^3d(a-b)\sqrt{a+b\cot^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/15*(Cos[c + d*x]^2*Cot[c + d*x]*(4*(a - b)^2*Cos[c + d*x]^2*Hypergeometr
ic2F1[2, 2, 7/2, ((a - b)*Cos[c + d*x]^2)/a]*(b + a*Tan[c + d*x]^2) - (15*a
*(2*b + 3*a*Tan[c + d*x]^2)*(ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*(b +
a*Tan[c + d*x]^2) - a*Sec[c + d*x]^2*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Ta
n[c + d*x]^2))/a^2]))/Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/
a^2]))/(a^3*(a - b)*d*Sqrt[a + b*Cot[c + d*x]^2])
```

fricas [B] time = 0.62, size = 526, normalized size = 6.19

$$\left[-\frac{\left(a^2+ab-\left(a^2-ab\right)\cos\left(2\,dx+2\,c\right)\right)\sqrt{-a+b}\,\log\left(-2\left(a^2-2\,ab+b^2\right)\cos\left(2\,dx+2\,c\right)^2+2\left(\left(a-b\right)\cos\left(2\,d\right.\right.\right.}{4\left(\left(a^4-3\,a^3b+\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a^2 + a*b - (a^2 - a*b)*cos(2*d*x + 2*c))*sqrt(-a + b)*log(-2*(a^2
- 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + 2*((a - b)*cos(2*d*x + 2*c) - b)*sqrt(-
a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(
2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x + 2*c)) + 4*(a*b - b^2
)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x
 + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^4 - a
^3*b - a^2*b^2 + a*b^3)*d), 1/2*((a^2 + a*b - (a^2 - a*b)*cos(2*d*x + 2*c))
*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(c
```

```
os(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) - b)) - 2*
(a*b - b^2)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))
*sin(2*d*x + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)
- (a^4 - a^3*b - a^2*b^2 + a*b^3)*d)]
```

giac [B] time = 10.14, size = 348, normalized size = 4.09

$$-\frac{\frac{\frac{\left(a^2b\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-2\,ab^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+b^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^4-3\,a^3b+3\,a^2b^2-ab^3}-\frac{a^2b\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-2\,ab^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+b^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4-3\,a^3b+3\,a^2b^2-ab^3}}{\sqrt{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(((a^2*b*sgn(tan(1/2*d*x + 1/2*c)) - 2*a*b^2*sgn(tan(1/2*d*x + 1/2*c)) + b
^3*sgn(tan(1/2*d*x + 1/2*c)))*tan(1/2*d*x + 1/2*c)^2/(a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3) - (a^2*b*sgn(tan(1/2*d*x + 1/2*c)) - 2*a*b^2*sgn(tan(1/2*d*x
+ 1/2*c)) + b^3*sgn(tan(1/2*d*x + 1/2*c)))/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b
^3))/sqrt(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1
/2*d*x + 1/2*c)^2 + b) + sqrt(b)*sgn(tan(1/2*d*x + 1/2*c))/(a^2 - a*b) - 2*
arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c)^4
 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(b))/
sqrt(a - b))/((a*sgn(tan(1/2*d*x + 1/2*c)) - b*sgn(tan(1/2*d*x + 1/2*c)))*s
qrt(a - b)))/d
```

maple [A] time = 0.36, size = 104, normalized size = 1.22

$$\frac{b\cot\left(dx+c\right)}{a\left(a-b\right)d\sqrt{a+b\left(\cot^2\left(dx+c\right)\right)}}-\frac{\sqrt{b^4\left(a-b\right)}\,\arctan\left(\frac{(a-b)b^2\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{d\left(a-b\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(3/2),x)
```

```
[Out] b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(1/2)-1/d/(a-b)^2*(b^4*(a-b))^(1/
2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+
c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cot\left(c+d\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*cot(c + d*x)^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cot^{2}\left(c + dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(-3/2), x)
```

3.36 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{b(5a-2b)\cot(c+dx)}{3a^2d(a-b)^2\sqrt{a+b\cot^2(c+dx)}}+\frac{b\cot(c+dx)}{3ad(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d(a-b)^{5/2}}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(5/2)/d+1/3*
b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(3/2)+1/3*(5*a-2*b)*b*cot(d*x+c)/
a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 414, 527, 12, 377, 203}

$$\frac{b(5a-2b)\cot(c+dx)}{3a^2d(a-b)^2\sqrt{a+b\cot^2(c+dx)}}+\frac{b\cot(c+dx)}{3ad(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/((a - b)^(5
/2)*d)) + (b*Cot[c + d*x])/(3*a*(a - b)*d*(a + b*Cot[c + d*x]^2)^(3/2)) + (
(5*a - 2*b)*b*Cot[c + d*x])/(3*a^2*(a - b)^2*d*Sqrt[a + b*Cot[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \cot(c+dx)\right)}{3a(a-b)d}$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}} - \frac{\text{Subst}}{}$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}} - \frac{\text{Subst}}{}$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}} - \frac{\text{Subst}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{5/2}d} + \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{(5a-2b)b}{3a^2(a-b)^2d\sqrt{a}}$$

Mathematica [C] time = 7.94, size = 367, normalized size = 2.72

$$-\frac{\cot^5(c+dx)\left(24(a-b)^3\cos^2(c+dx)\left(a\tan^2(c+dx)+b\right)^2 {}_3F_2\left(2,2,2;1,\frac{9}{2};\frac{(a-b)\cos^2(c+dx)}{a}\right)+24(a-b)^3\cos^2(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-5/2),x]
```

```
[Out] -1/315*(Cot[c + d*x]^5*(24*(a - b)^3*Cos[c + d*x]^2*HypergeometricPFQ[{2, 2
, 2}, {1, 9/2}, ((a - b)*Cos[c + d*x]^2)/a]*(b + a*Tan[c + d*x]^2)^2 + 24*(
a - b)^3*Cos[c + d*x]^2*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[c + d*x]^
2)/a]*(3*b^2 + 7*a*b*Tan[c + d*x]^2 + 4*a^2*Tan[c + d*x]^4) - (35*a*(8*b^2
+ 20*a*b*Tan[c + d*x]^2 + 15*a^2*Tan[c + d*x]^4)*(-3*ArcSin[Sqrt[((a - b)*C
os[c + d*x]^2)/a]]*(b + a*Tan[c + d*x]^2)^2 + a*Sec[c + d*x]^2*Sqrt[((a - b
)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2]*(4*b + a*(-1 + 3*Tan[c + d*x]
^2))))/Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2]))/(a^5*(a
- b)^2*d*(1 + Cot[c + d*x]^2)*Sqrt[a + b*Cot[c + d*x]^2]*(1 + (b*Cot[c + d*
x]^2)/a))
```

fricas [B] time = 0.68, size = 898, normalized size = 6.65

$$\left[-\frac{3\left(a^4+2\,a^3b+a^2b^2+\left(a^4-2\,a^3b+a^2b^2\right)\cos\left(2\,dx+2\,c\right)^2-2\left(a^4-a^2b^2\right)\cos\left(2\,dx+2\,c\right)\right)\sqrt{-a+b}\,\log\left(-2\left(a\right.\right.}{12\left(\left(a^7-5\,a^6\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b + a^2*b^2)*cos(2*d*x +
2*c)^2 - 2*(a^4 - a^2*b^2)*cos(2*d*x + 2*c))*sqrt(-a + b)*log(-2*(a^2 - 2*a
*b + b^2)*cos(2*d*x + 2*c)^2 - 2*((a - b)*cos(2*d*x + 2*c) - b)*sqrt(-a + b
)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x
 + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x + 2*c)) - 8*(3*a^3*b - 2*a^
2*b^2 - 2*a*b^3 + b^4 - (3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x + 2
*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*
d*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5
)*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a^3*b
^4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a
^3*b^4 - a^2*b^5)*d), -1/6*(3*(a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b + a
^2*b^2)*cos(2*d*x + 2*c)^2 - 2*(a^4 - a^2*b^2)*cos(2*d*x + 2*c))*sqrt(a - b
)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x +
2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) - b)) - 4*(3*a^3*b -
2*a^2*b^2 - 2*a*b^3 + b^4 - (3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x
 + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*si
n(2*d*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2
*b^5)*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a
^3*b^4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3
 + a^3*b^4 - a^2*b^5)*d)]
```

giac [B] time = 13.77, size = 1341, normalized size = 9.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((5*a*b - 2*b^2)*sgn(tan(1/2*d*x + 1/2*c))/(a^4*sqrt(b) - 2*a^3*b^(3/2
) + a^2*b^(5/2)) + ((((5*a^9*b^2*sgn(tan(1/2*d*x + 1/2*c)) - 42*a^8*b^3*sgn
(tan(1/2*d*x + 1/2*c)) + 156*a^7*b^4*sgn(tan(1/2*d*x + 1/2*c)) - 336*a^6*b^
5*sgn(tan(1/2*d*x + 1/2*c)) + 462*a^5*b^6*sgn(tan(1/2*d*x + 1/2*c)) - 420*a
^4*b^7*sgn(tan(1/2*d*x + 1/2*c)) + 252*a^3*b^8*sgn(tan(1/2*d*x + 1/2*c)) -
96*a^2*b^9*sgn(tan(1/2*d*x + 1/2*c)) + 21*a*b^10*sgn(tan(1/2*d*x + 1/2*c))
- 2*b^11*sgn(tan(1/2*d*x + 1/2*c)))*tan(1/2*d*x + 1/2*c)^2/(a^12 - 10*a^11*
b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 1
```

```
20*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10) + 3*(8*a^10*b*sgn(tan(1/2*
d*x + 1/2*c)) - 73*a^9*b^2*sgn(tan(1/2*d*x + 1/2*c)) + 298*a^8*b^3*sgn(tan(
1/2*d*x + 1/2*c)) - 716*a^7*b^4*sgn(tan(1/2*d*x + 1/2*c)) + 1120*a^6*b^5*sg
n(tan(1/2*d*x + 1/2*c)) - 1190*a^5*b^6*sgn(tan(1/2*d*x + 1/2*c)) + 868*a^4*
b^7*sgn(tan(1/2*d*x + 1/2*c)) - 428*a^3*b^8*sgn(tan(1/2*d*x + 1/2*c)) + 136
*a^2*b^9*sgn(tan(1/2*d*x + 1/2*c)) - 25*a*b^10*sgn(tan(1/2*d*x + 1/2*c)) +
2*b^11*sgn(tan(1/2*d*x + 1/2*c)))/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a^9
*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8 -
 10*a^3*b^9 + a^2*b^10))*tan(1/2*d*x + 1/2*c)^2 - 3*(8*a^10*b*sgn(tan(1/2*d
*x + 1/2*c)) - 73*a^9*b^2*sgn(tan(1/2*d*x + 1/2*c)) + 298*a^8*b^3*sgn(tan(1
/2*d*x + 1/2*c)) - 716*a^7*b^4*sgn(tan(1/2*d*x + 1/2*c)) + 1120*a^6*b^5*sgn
(tan(1/2*d*x + 1/2*c)) - 1190*a^5*b^6*sgn(tan(1/2*d*x + 1/2*c)) + 868*a^4*b
^7*sgn(tan(1/2*d*x + 1/2*c)) - 428*a^3*b^8*sgn(tan(1/2*d*x + 1/2*c)) + 136*
a^2*b^9*sgn(tan(1/2*d*x + 1/2*c)) - 25*a*b^10*sgn(tan(1/2*d*x + 1/2*c)) + 2
*b^11*sgn(tan(1/2*d*x + 1/2*c)))/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a^9*
b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8 -
10*a^3*b^9 + a^2*b^10))*tan(1/2*d*x + 1/2*c)^2 - (5*a^9*b^2*sgn(tan(1/2*d*x
 + 1/2*c)) - 42*a^8*b^3*sgn(tan(1/2*d*x + 1/2*c)) + 156*a^7*b^4*sgn(tan(1/2
*d*x + 1/2*c)) - 336*a^6*b^5*sgn(tan(1/2*d*x + 1/2*c)) + 462*a^5*b^6*sgn(ta
n(1/2*d*x + 1/2*c)) - 420*a^4*b^7*sgn(tan(1/2*d*x + 1/2*c)) + 252*a^3*b^8*s
gn(tan(1/2*d*x + 1/2*c)) - 96*a^2*b^9*sgn(tan(1/2*d*x + 1/2*c)) + 21*a*b^10
*sgn(tan(1/2*d*x + 1/2*c)) - 2*b^11*sgn(tan(1/2*d*x + 1/2*c)))/(a^12 - 10*a
^11*b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6
 - 120*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10))/(b*tan(1/2*d*x + 1/2*
c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b)^(3/2) -
 6*arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c
)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(b
))/sqrt(a - b))/((a^2*sgn(tan(1/2*d*x + 1/2*c)) - 2*a*b*sgn(tan(1/2*d*x + 1
/2*c)) + b^2*sgn(tan(1/2*d*x + 1/2*c)))*sqrt(a - b)))/d
```

maple [A] time = 0.38, size = 176, normalized size = 1.30

$$\frac{b\cot(dx+c)}{d(a-b)^2 a\sqrt{a+b\left(\cot^2(dx+c)\right)}}+\frac{b\cot(dx+c)}{3a(a-b)d\left(a+b\left(\cot^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{2b\cot(dx+c)}{3d(a-b)a^2\sqrt{a+b\left(\cot^2(dx+c)\right)}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/d*b/(a-b)^2*cot(d*x+c)/a/(a+b*cot(d*x+c)^2)^(1/2)+1/3*b*cot(d*x+c)/a/(a-b
)/d/(a+b*cot(d*x+c)^2)^(3/2)+2/3/d*b/(a-b)/a^2*cot(d*x+c)/(a+b*cot(d*x+c)^2
)^(1/2)-1/d/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2
)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cot(c+d\,x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*cot(c + d*x)^2)^(5/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cot^2\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(-5/2), x)
```

3.37 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=190

$$\frac{b(9a-4b)\cot(c+dx)}{15a^2d(a-b)^2\left(a+b\cot^2(c+dx)\right)^{3/2}}+\frac{b\left(33a^2-26ab+8b^2\right)\cot(c+dx)}{15a^3d(a-b)^3\sqrt{a+b\cot^2(c+dx)}}+\frac{b\cot(c+dx)}{5ad(a-b)\left(a+b\cot^2(c+dx)\right)^{5/2}}-\text{t}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(7/2)/d+1/5*
b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(5/2)+1/15*(9*a-4*b)*b*cot(d*x+c)
/a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)^(3/2)+1/15*b*(33*a^2-26*a*b+8*b^2)*cot(d*
x+c)/a^3/(a-b)^3/d/(a+b*cot(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 414, 527, 12, 377, 203}

$$\frac{b\left(33a^2-26ab+8b^2\right)\cot(c+dx)}{15a^3d(a-b)^3\sqrt{a+b\cot^2(c+dx)}}+\frac{b(9a-4b)\cot(c+dx)}{15a^2d(a-b)^2\left(a+b\cot^2(c+dx)\right)^{3/2}}+\frac{b\cot(c+dx)}{5ad(a-b)\left(a+b\cot^2(c+dx)\right)^{5/2}}-\text{t}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-7/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/((a - b)^(7
/2)*d)) + (b*Cot[c + d*x])/(5*a*(a - b)*d*(a + b*Cot[c + d*x]^2)^(5/2)) + (
(9*a - 4*b)*b*Cot[c + d*x])/(15*a^2*(a - b)^2*d*(a + b*Cot[c + d*x]^2)^(3/2
)) + (b*(33*a^2 - 26*a*b + 8*b^2)*Cot[c + d*x])/(15*a^3*(a - b)^3*d*Sqrt[a
+ b*Cot[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{7/2}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{5a-4b-4bx^2}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \cot(c+dx)\right)}{5a(a-b)d}$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} - \text{Subst}$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{b\,(33a}{15a^3(}$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{b\,(33a}{15a^3(}$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{b\,(33a}{15a^3(}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{7/2}d} + \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b}{15a^2(a-b)^2d\,(a+}$$

Mathematica [C] time = 14.68, size = 2553, normalized size = 13.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-7/2),x]
```

```
[Out] -1/4725*(Cot[c + d*x]*(-33075*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]] + (9
9225*(a - b)*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2)/a - (
99225*(a - b)^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^4)/a^
2 + (33075*(a - b)^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^
6)/a^3 - (66150*b*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cot[c + d*x]^2)/
a + (198450*(a - b)*b*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]
^2*Cot[c + d*x]^2)/a^2 + (66150*(a - b)^3*b*ArcSin[Sqrt[((a - b)*Cos[c + d*
x]^2)/a]]*Cos[c + d*x]^6*Cot[c + d*x]^2)/a^4 - (52920*b^2*ArcSin[Sqrt[((a -
 b)*Cos[c + d*x]^2)/a]]*Cot[c + d*x]^4)/a^2 + (158760*(a - b)*b^2*ArcSin[Sq
rt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2*Cot[c + d*x]^4)/a^3 - (15876
0*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^4*Cot
[c + d*x]^4)/a^4 + (52920*(a - b)^3*b^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2
)/a]]*Cos[c + d*x]^6*Cot[c + d*x]^4)/a^5 - (15120*b^3*ArcSin[Sqrt[((a - b)*
Cos[c + d*x]^2)/a]]*Cot[c + d*x]^6)/a^3 + (45360*(a - b)*b^3*ArcSin[Sqrt[((
a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2*Cot[c + d*x]^6)/a^4 - (45360*(a -
 b)^2*b^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^4*Cot[c + d
*x]^6)/a^5 + (15120*(a - b)^3*b^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*
Cos[c + d*x]^6*Cot[c + d*x]^6)/a^6 - 77175*(((a - b)*Cos[c + d*x]^2)/a)^(3/
2)*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a] + 50715*(((a - b)*Cos[c
+ d*x]^2)/a)^(5/2)*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a] - (15435
0*b*(((a - b)*Cos[c + d*x]^2)/a)^(3/2)*Cot[c + d*x]^2*Sqrt[(Cos[c + d*x]^2*
(b + a*Tan[c + d*x]^2))/a])/a + (101430*b*(((a - b)*Cos[c + d*x]^2)/a)^(5/2
)*Cot[c + d*x]^2*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a - (1234
80*b^2*(((a - b)*Cos[c + d*x]^2)/a)^(3/2)*Cot[c + d*x]^4*Sqrt[(Cos[c + d*x]
^2*(b + a*Tan[c + d*x]^2))/a])/a^2 + (81144*b^2*(((a - b)*Cos[c + d*x]^2)/a
)^(5/2)*Cot[c + d*x]^4*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^2
 - (35280*b^3*(((a - b)*Cos[c + d*x]^2)/a)^(3/2)*Cot[c + d*x]^6*Sqrt[(Cos[c
 + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^3 + (23184*b^3*(((a - b)*Cos[c + d*
x]^2)/a)^(5/2)*Cot[c + d*x]^6*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/
a])/a^3 + 1420*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Hypergeometric2F1[2, 2, 1
1/2, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2
))/a] + (3540*b*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^2*Hypergeom
etric2F1[2, 2, 11/2, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b +
a*Tan[c + d*x]^2))/a])/a + (3000*b^2*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot
[c + d*x]^4*Hypergeometric2F1[2, 2, 11/2, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[
(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^2 + (880*b^3*(((a - b)*Cos[c
+ d*x]^2)/a)^(9/2)*Cot[c + d*x]^6*Hypergeometric2F1[2, 2, 11/2, ((a - b)*Co
s[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^3 + 600
*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*HypergeometricPFQ[{2, 2, 2}, {1, 11/2},
 ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a
] + (1680*b*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^2*Hypergeometri
cPFQ[{2, 2, 2}, {1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2
*(b + a*Tan[c + d*x]^2))/a])/a + (1560*b^2*(((a - b)*Cos[c + d*x]^2)/a)^(9/
2)*Cot[c + d*x]^4*HypergeometricPFQ[{2, 2, 2}, {1, 11/2}, ((a - b)*Cos[c +
d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^2 + (480*b^3*
(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^6*HypergeometricPFQ[{2, 2,
2}, {1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[
c + d*x]^2))/a])/a^3 + 80*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Hypergeometric
PFQ[{2, 2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d
*x]^2*(b + a*Tan[c + d*x]^2))/a] + (240*b*(((a - b)*Cos[c + d*x]^2)/a)^(9/2
)*Cot[c + d*x]^2*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos
[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a + (240*b
^2*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^4*HypergeometricPFQ[{2,
2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b
 + a*Tan[c + d*x]^2))/a])/a^2 + (80*b^3*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*
Cot[c + d*x]^6*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos[c
 + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^3 + 33075*
Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2] + (66150*b*Cot[c
```

```
+ d*x]^2*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2])/a + (52
920*b^2*Cot[c + d*x]^4*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))
/a^2])/a^2 + (15120*b^3*Cot[c + d*x]^6*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*
Tan[c + d*x]^2))/a^2])/a^3 - (198450*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Cos[c
 + d*x]^2)/a]])/(a^3*(Tan[c + d*x] + Tan[c + d*x]^3)^2)))/(a^3*d*(((a - b)*
Cos[c + d*x]^2)/a)^(7/2)*(1 + Cot[c + d*x]^2)*Sqrt[a + b*Cot[c + d*x]^2]*(1
 + (b*Cot[c + d*x]^2)/a)^2*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])
```

fricas [B] time = 0.63, size = 1452, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 - 3*a^5*b + 3*a^4*b^
2 - a^3*b^3)*cos(2*d*x + 2*c)^3 + 3*(a^6 - a^5*b - a^4*b^2 + a^3*b^3)*cos(2
*d*x + 2*c)^2 - 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*cos(2*d*x + 2*c))*sqrt(
-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + 2*((a - b)*cos(2*d*
x + 2*c) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d
*x + 2*c) - 1))*sin(2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x +
2*c)) + 4*(45*a^5*b - 15*a^4*b^2 - 47*a^3*b^3 + 11*a^2*b^4 + 14*a*b^5 - 8*b
^6 + (45*a^5*b - 165*a^4*b^2 + 233*a^3*b^3 - 159*a^2*b^4 + 54*a*b^5 - 8*b^6
)*cos(2*d*x + 2*c)^2 - 2*(45*a^5*b - 90*a^4*b^2 + 27*a^3*b^3 + 44*a^2*b^4 -
 34*a*b^5 + 8*b^6)*cos(2*d*x + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b
)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c))/((a^10 - 7*a^9*b + 21*a^8*b^2 -
 35*a^7*b^3 + 35*a^6*b^4 - 21*a^5*b^5 + 7*a^4*b^6 - a^3*b^7)*d*cos(2*d*x +
2*c)^3 - 3*(a^10 - 5*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 - 5*a^6*b^4 + 9*a^5*b^5
- 5*a^4*b^6 + a^3*b^7)*d*cos(2*d*x + 2*c)^2 + 3*(a^10 - 3*a^9*b + a^8*b^2 +
 5*a^7*b^3 - 5*a^6*b^4 - a^5*b^5 + 3*a^4*b^6 - a^3*b^7)*d*cos(2*d*x + 2*c)
- (a^10 - a^9*b - 3*a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 - 3*a^5*b^5 - a^4*b^6 +
 a^3*b^7)*d), 1/30*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 - 3*a^5*
b + 3*a^4*b^2 - a^3*b^3)*cos(2*d*x + 2*c)^3 + 3*(a^6 - a^5*b - a^4*b^2 + a^
3*b^3)*cos(2*d*x + 2*c)^2 - 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*cos(2*d*x +
 2*c))*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a -
 b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) - b)
) - 2*(45*a^5*b - 15*a^4*b^2 - 47*a^3*b^3 + 11*a^2*b^4 + 14*a*b^5 - 8*b^6 +
 (45*a^5*b - 165*a^4*b^2 + 233*a^3*b^3 - 159*a^2*b^4 + 54*a*b^5 - 8*b^6)*co
s(2*d*x + 2*c)^2 - 2*(45*a^5*b - 90*a^4*b^2 + 27*a^3*b^3 + 44*a^2*b^4 - 34*
a*b^5 + 8*b^6)*cos(2*d*x + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(c
os(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c))/((a^10 - 7*a^9*b + 21*a^8*b^2 - 35*
a^7*b^3 + 35*a^6*b^4 - 21*a^5*b^5 + 7*a^4*b^6 - a^3*b^7)*d*cos(2*d*x + 2*c)
^3 - 3*(a^10 - 5*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 - 5*a^6*b^4 + 9*a^5*b^5 - 5*
a^4*b^6 + a^3*b^7)*d*cos(2*d*x + 2*c)^2 + 3*(a^10 - 3*a^9*b + a^8*b^2 + 5*a
^7*b^3 - 5*a^6*b^4 - a^5*b^5 + 3*a^4*b^6 - a^3*b^7)*d*cos(2*d*x + 2*c) - (a
^10 - a^9*b - 3*a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 - 3*a^5*b^5 - a^4*b^6 + a^3
*b^7)*d)]
```

giac [B] time = 31.53, size = 3719, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*((33*a^2*b - 26*a*b^2 + 8*b^3)*sgn(tan(1/2*d*x + 1/2*c))/(a^6*sqrt(b)
 - 3*a^5*b^(3/2) + 3*a^4*b^(5/2) - a^3*b^(7/2)) - 30*arctan(-1/2*(sqrt(b)*t
an(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1
/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(b))/sqrt(a - b))/((a^3*sgn
(tan(1/2*d*x + 1/2*c)) - 3*a^2*b*sgn(tan(1/2*d*x + 1/2*c)) + 3*a*b^2*sgn(ta
```

```
n(1/2*d*x + 1/2*c)) - b^3*sgn(tan(1/2*d*x + 1/2*c)))*sqrt(a - b)) + ((((((3
3*a^20*b^3*sgn(tan(1/2*d*x + 1/2*c)) - 620*a^19*b^4*sgn(tan(1/2*d*x + 1/2*c
)) + 5525*a^18*b^5*sgn(tan(1/2*d*x + 1/2*c)) - 31050*a^17*b^6*sgn(tan(1/2*d
*x + 1/2*c)) + 123420*a^16*b^7*sgn(tan(1/2*d*x + 1/2*c)) - 368832*a^15*b^8*
sgn(tan(1/2*d*x + 1/2*c)) + 859860*a^14*b^9*sgn(tan(1/2*d*x + 1/2*c)) - 160
1400*a^13*b^10*sgn(tan(1/2*d*x + 1/2*c)) + 2419950*a^12*b^11*sgn(tan(1/2*d*
x + 1/2*c)) - 2996760*a^11*b^12*sgn(tan(1/2*d*x + 1/2*c)) + 3058198*a^10*b^
13*sgn(tan(1/2*d*x + 1/2*c)) - 2576860*a^9*b^14*sgn(tan(1/2*d*x + 1/2*c)) +
 1790100*a^8*b^15*sgn(tan(1/2*d*x + 1/2*c)) - 1020000*a^7*b^16*sgn(tan(1/2*
d*x + 1/2*c)) + 472260*a^6*b^17*sgn(tan(1/2*d*x + 1/2*c)) - 175032*a^5*b^18
*sgn(tan(1/2*d*x + 1/2*c)) + 50745*a^4*b^19*sgn(tan(1/2*d*x + 1/2*c)) - 111
00*a^3*b^20*sgn(tan(1/2*d*x + 1/2*c)) + 1725*a^2*b^21*sgn(tan(1/2*d*x + 1/2
*c)) - 170*a*b^22*sgn(tan(1/2*d*x + 1/2*c)) + 8*b^23*sgn(tan(1/2*d*x + 1/2*
c)))*tan(1/2*d*x + 1/2*c)^2/(a^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^
3 + 5985*a^20*b^4 - 20349*a^19*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 + 203
490*a^16*b^8 - 293930*a^15*b^9 + 352716*a^14*b^10 - 352716*a^13*b^11 + 2939
30*a^12*b^12 - 203490*a^11*b^13 + 116280*a^10*b^14 - 54264*a^9*b^15 + 20349
*a^8*b^16 - 5985*a^7*b^17 + 1330*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^
3*b^21) + 5*(60*a^21*b^2*sgn(tan(1/2*d*x + 1/2*c)) - 1165*a^20*b^3*sgn(tan(
1/2*d*x + 1/2*c)) + 10752*a^19*b^4*sgn(tan(1/2*d*x + 1/2*c)) - 62729*a^18*b
^5*sgn(tan(1/2*d*x + 1/2*c)) + 259530*a^17*b^6*sgn(tan(1/2*d*x + 1/2*c)) -
809676*a^16*b^7*sgn(tan(1/2*d*x + 1/2*c)) + 1977168*a^15*b^8*sgn(tan(1/2*d*
x + 1/2*c)) - 3871716*a^14*b^9*sgn(tan(1/2*d*x + 1/2*c)) + 6178752*a^13*b^1
0*sgn(tan(1/2*d*x + 1/2*c)) - 8121750*a^12*b^11*sgn(tan(1/2*d*x + 1/2*c)) +
 8850608*a^11*b^12*sgn(tan(1/2*d*x + 1/2*c)) - 8020974*a^10*b^13*sgn(tan(1/
2*d*x + 1/2*c)) + 6045676*a^9*b^14*sgn(tan(1/2*d*x + 1/2*c)) - 3778692*a^8*
b^15*sgn(tan(1/2*d*x + 1/2*c)) + 1946160*a^7*b^16*sgn(tan(1/2*d*x + 1/2*c))
 - 817428*a^6*b^17*sgn(tan(1/2*d*x + 1/2*c)) + 275604*a^5*b^18*sgn(tan(1/2*
d*x + 1/2*c)) - 72837*a^4*b^19*sgn(tan(1/2*d*x + 1/2*c)) + 14544*a^3*b^20*s
gn(tan(1/2*d*x + 1/2*c)) - 2065*a^2*b^21*sgn(tan(1/2*d*x + 1/2*c)) + 186*a*
b^22*sgn(tan(1/2*d*x + 1/2*c)) - 8*b^23*sgn(tan(1/2*d*x + 1/2*c)))/(a^24 -
21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^3 + 5985*a^20*b^4 - 20349*a^19*b^5 +
 54264*a^18*b^6 - 116280*a^17*b^7 + 203490*a^16*b^8 - 293930*a^15*b^9 + 352
716*a^14*b^10 - 352716*a^13*b^11 + 293930*a^12*b^12 - 203490*a^11*b^13 + 11
6280*a^10*b^14 - 54264*a^9*b^15 + 20349*a^8*b^16 - 5985*a^7*b^17 + 1330*a^6
*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^3*b^21))*tan(1/2*d*x + 1/2*c)^2 + 10
*(72*a^22*b*sgn(tan(1/2*d*x + 1/2*c)) - 1458*a^21*b^2*sgn(tan(1/2*d*x + 1/2
*c)) + 14067*a^20*b^3*sgn(tan(1/2*d*x + 1/2*c)) - 86018*a^19*b^4*sgn(tan(1/
2*d*x + 1/2*c)) + 374075*a^18*b^5*sgn(tan(1/2*d*x + 1/2*c)) - 1230570*a^17*
b^6*sgn(tan(1/2*d*x + 1/2*c)) + 3179748*a^16*b^7*sgn(tan(1/2*d*x + 1/2*c))
- 6614904*a^15*b^8*sgn(tan(1/2*d*x + 1/2*c)) + 11265084*a^14*b^9*sgn(tan(1/
2*d*x + 1/2*c)) - 15882420*a^13*b^10*sgn(tan(1/2*d*x + 1/2*c)) + 18674058*a
^12*b^11*sgn(tan(1/2*d*x + 1/2*c)) - 18386316*a^11*b^12*sgn(tan(1/2*d*x + 1
/2*c)) + 15180490*a^10*b^13*sgn(tan(1/2*d*x + 1/2*c)) - 10497364*a^9*b^14*s
gn(tan(1/2*d*x + 1/2*c)) + 6055740*a^8*b^15*sgn(tan(1/2*d*x + 1/2*c)) - 289
3944*a^7*b^16*sgn(tan(1/2*d*x + 1/2*c)) + 1133220*a^6*b^17*sgn(tan(1/2*d*x
+ 1/2*c)) - 357786*a^5*b^18*sgn(tan(1/2*d*x + 1/2*c)) + 88923*a^4*b^19*sgn(
tan(1/2*d*x + 1/2*c)) - 16770*a^3*b^20*sgn(tan(1/2*d*x + 1/2*c)) + 2259*a^2
*b^21*sgn(tan(1/2*d*x + 1/2*c)) - 194*a*b^22*sgn(tan(1/2*d*x + 1/2*c)) + 8*
b^23*sgn(tan(1/2*d*x + 1/2*c)))/(a^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^2
1*b^3 + 5985*a^20*b^4 - 20349*a^19*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 +
 203490*a^16*b^8 - 293930*a^15*b^9 + 352716*a^14*b^10 - 352716*a^13*b^11 +
293930*a^12*b^12 - 203490*a^11*b^13 + 116280*a^10*b^14 - 54264*a^9*b^15 + 2
0349*a^8*b^16 - 5985*a^7*b^17 + 1330*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20
- a^3*b^21))*tan(1/2*d*x + 1/2*c)^2 - 10*(72*a^22*b*sgn(tan(1/2*d*x + 1/2*c
)) - 1458*a^21*b^2*sgn(tan(1/2*d*x + 1/2*c)) + 14067*a^20*b^3*sgn(tan(1/2*d
*x + 1/2*c)) - 86018*a^19*b^4*sgn(tan(1/2*d*x + 1/2*c)) + 374075*a^18*b^5*s
gn(tan(1/2*d*x + 1/2*c)) - 1230570*a^17*b^6*sgn(tan(1/2*d*x + 1/2*c)) + 317
9748*a^16*b^7*sgn(tan(1/2*d*x + 1/2*c)) - 6614904*a^15*b^8*sgn(tan(1/2*d*x
```

```
+ 1/2*c)) + 11265084*a^14*b^9*sgn(tan(1/2*d*x + 1/2*c)) - 15882420*a^13*b^1
0*sgn(tan(1/2*d*x + 1/2*c)) + 18674058*a^12*b^11*sgn(tan(1/2*d*x + 1/2*c))
- 18386316*a^11*b^12*sgn(tan(1/2*d*x + 1/2*c)) + 15180490*a^10*b^13*sgn(tan
(1/2*d*x + 1/2*c)) - 10497364*a^9*b^14*sgn(tan(1/2*d*x + 1/2*c)) + 6055740*
a^8*b^15*sgn(tan(1/2*d*x + 1/2*c)) - 2893944*a^7*b^16*sgn(tan(1/2*d*x + 1/2
*c)) + 1133220*a^6*b^17*sgn(tan(1/2*d*x + 1/2*c)) - 357786*a^5*b^18*sgn(tan
(1/2*d*x + 1/2*c)) + 88923*a^4*b^19*sgn(tan(1/2*d*x + 1/2*c)) - 16770*a^3*b
^20*sgn(tan(1/2*d*x + 1/2*c)) + 2259*a^2*b^21*sgn(tan(1/2*d*x + 1/2*c)) - 1
94*a*b^22*sgn(tan(1/2*d*x + 1/2*c)) + 8*b^23*sgn(tan(1/2*d*x + 1/2*c)))/(a^
24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^3 + 5985*a^20*b^4 - 20349*a^19*
b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 + 203490*a^16*b^8 - 293930*a^15*b^9
+ 352716*a^14*b^10 - 352716*a^13*b^11 + 293930*a^12*b^12 - 203490*a^11*b^13
 + 116280*a^10*b^14 - 54264*a^9*b^15 + 20349*a^8*b^16 - 5985*a^7*b^17 + 133
0*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^3*b^21))*tan(1/2*d*x + 1/2*c)^2
 - 5*(60*a^21*b^2*sgn(tan(1/2*d*x + 1/2*c)) - 1165*a^20*b^3*sgn(tan(1/2*d*x
 + 1/2*c)) + 10752*a^19*b^4*sgn(tan(1/2*d*x + 1/2*c)) - 62729*a^18*b^5*sgn(
tan(1/2*d*x + 1/2*c)) + 259530*a^17*b^6*sgn(tan(1/2*d*x + 1/2*c)) - 809676*
a^16*b^7*sgn(tan(1/2*d*x + 1/2*c)) + 1977168*a^15*b^8*sgn(tan(1/2*d*x + 1/2
*c)) - 3871716*a^14*b^9*sgn(tan(1/2*d*x + 1/2*c)) + 6178752*a^13*b^10*sgn(t
an(1/2*d*x + 1/2*c)) - 8121750*a^12*b^11*sgn(tan(1/2*d*x + 1/2*c)) + 885060
8*a^11*b^12*sgn(tan(1/2*d*x + 1/2*c)) - 8020974*a^10*b^13*sgn(tan(1/2*d*x +
 1/2*c)) + 6045676*a^9*b^14*sgn(tan(1/2*d*x + 1/2*c)) - 3778692*a^8*b^15*sg
n(tan(1/2*d*x + 1/2*c)) + 1946160*a^7*b^16*sgn(tan(1/2*d*x + 1/2*c)) - 8174
28*a^6*b^17*sgn(tan(1/2*d*x + 1/2*c)) + 275604*a^5*b^18*sgn(tan(1/2*d*x + 1
/2*c)) - 72837*a^4*b^19*sgn(tan(1/2*d*x + 1/2*c)) + 14544*a^3*b^20*sgn(tan(
1/2*d*x + 1/2*c)) - 2065*a^2*b^21*sgn(tan(1/2*d*x + 1/2*c)) + 186*a*b^22*sg
n(tan(1/2*d*x + 1/2*c)) - 8*b^23*sgn(tan(1/2*d*x + 1/2*c)))/(a^24 - 21*a^23
*b + 210*a^22*b^2 - 1330*a^21*b^3 + 5985*a^20*b^4 - 20349*a^19*b^5 + 54264*
a^18*b^6 - 116280*a^17*b^7 + 203490*a^16*b^8 - 293930*a^15*b^9 + 352716*a^1
4*b^10 - 352716*a^13*b^11 + 293930*a^12*b^12 - 203490*a^11*b^13 + 116280*a^
10*b^14 - 54264*a^9*b^15 + 20349*a^8*b^16 - 5985*a^7*b^17 + 1330*a^6*b^18 -
 210*a^5*b^19 + 21*a^4*b^20 - a^3*b^21))*tan(1/2*d*x + 1/2*c)^2 - (33*a^20*
b^3*sgn(tan(1/2*d*x + 1/2*c)) - 620*a^19*b^4*sgn(tan(1/2*d*x + 1/2*c)) + 55
25*a^18*b^5*sgn(tan(1/2*d*x + 1/2*c)) - 31050*a^17*b^6*sgn(tan(1/2*d*x + 1/
2*c)) + 123420*a^16*b^7*sgn(tan(1/2*d*x + 1/2*c)) - 368832*a^15*b^8*sgn(tan
(1/2*d*x + 1/2*c)) + 859860*a^14*b^9*sgn(tan(1/2*d*x + 1/2*c)) - 1601400*a^
13*b^10*sgn(tan(1/2*d*x + 1/2*c)) + 2419950*a^12*b^11*sgn(tan(1/2*d*x + 1/2
*c)) - 2996760*a^11*b^12*sgn(tan(1/2*d*x + 1/2*c)) + 3058198*a^10*b^13*sgn(
tan(1/2*d*x + 1/2*c)) - 2576860*a^9*b^14*sgn(tan(1/2*d*x + 1/2*c)) + 179010
0*a^8*b^15*sgn(tan(1/2*d*x + 1/2*c)) - 1020000*a^7*b^16*sgn(tan(1/2*d*x + 1
/2*c)) + 472260*a^6*b^17*sgn(tan(1/2*d*x + 1/2*c)) - 175032*a^5*b^18*sgn(ta
n(1/2*d*x + 1/2*c)) + 50745*a^4*b^19*sgn(tan(1/2*d*x + 1/2*c)) - 11100*a^3*
b^20*sgn(tan(1/2*d*x + 1/2*c)) + 1725*a^2*b^21*sgn(tan(1/2*d*x + 1/2*c)) -
170*a*b^22*sgn(tan(1/2*d*x + 1/2*c)) + 8*b^23*sgn(tan(1/2*d*x + 1/2*c)))/(a
^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^3 + 5985*a^20*b^4 - 20349*a^19
*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 + 203490*a^16*b^8 - 293930*a^15*b^9
 + 352716*a^14*b^10 - 352716*a^13*b^11 + 293930*a^12*b^12 - 203490*a^11*b^1
3 + 116280*a^10*b^14 - 54264*a^9*b^15 + 20349*a^8*b^16 - 5985*a^7*b^17 + 13
30*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^3*b^21))/(b*tan(1/2*d*x + 1/2*
c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b)^(5/2))/
d
```

maple [A] time = 0.38, size = 284, normalized size = 1.49

$$\frac{b\cot(dx+c)}{5a(a-b)d\left(a+b\left(\cot^2(dx+c)\right)\right)^{\frac{5}{2}}}+\frac{4b\cot(dx+c)}{15d(a-b)a^2\left(a+b\left(\cot^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{8b\cot(dx+c)}{15d(a-b)a^3\sqrt{a+b\left(\cot^2(dx+c)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(7/2),x)
```

```
[Out] 1/5*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(5/2)+4/15/d*b/(a-b)/a^2*cot(
d*x+c)/(a+b*cot(d*x+c)^2)^(3/2)+8/15/d*b/(a-b)/a^3*cot(d*x+c)/(a+b*cot(d*x+
c)^2)^(1/2)+1/d*b/(a-b)^3*cot(d*x+c)/a/(a+b*cot(d*x+c)^2)^(1/2)+1/3/d*b/(a-
b)^2*cot(d*x+c)/a/(a+b*cot(d*x+c)^2)^(3/2)+2/3/d*b/(a-b)^2/a^2*cot(d*x+c)/(
a+b*cot(d*x+c)^2)^(1/2)-1/d/(a-b)^4*(b^4*(a-b))^(1/2)/b^2*arctan((a-b)*b^2/
(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{cot}\left(c+d\,x\right)}^2+a\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a + b*cot(c + d*x)^2)^(7/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \cot^{2}{\left(c + dx \right)}\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(-7/2), x)
```

3.38 $\int \left(1-\cot^2(x)\right)^{3/2} dx$

Optimal. Leaf size=54

$$\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{5}{2}\sin^{-1}(\cot(x))$$

```
[Out] 5/2*arcsin(cot(x))-2*arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)+1/2*
cot(x)*(1-cot(x)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3661, 416, 523, 216, 377, 203}

$$\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{5}{2}\sin^{-1}(\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cot[x]^2)^(3/2),x]
```

```
[Out] (5*ArcSin[Cot[x]])/2 - 2*Sqrt[2]*ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]
] + (Cot[x]*Sqrt[1 - Cot[x]^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \left(1-\cot^2(x)\right)^{3/2}\,dx &= -\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{1+x^2}\,dx,x,\cot(x)\right)\\ &= \frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}-\frac{1}{2}\text{Subst}\left(\int \frac{3-5x^2}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx,x,\cot(x)\right)\\ &= \frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}+\frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,\cot(x)\right)-4\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(1\right.\\ &= \frac{5}{2}\sin^{-1}(\cot(x))+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}-4\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{\cot(x)}{\sqrt{1-\cot^2(x)}}\right)\\ &= \frac{5}{2}\sin^{-1}(\cot(x))-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}\end{aligned}$$

Mathematica [B] time = 0.39, size = 123, normalized size = 2.28

$$\frac{1}{2}\left(1-\cot^2(x)\right)^{3/2}\sec^2(2x)\left(-\frac{1}{4}\sin(4x)-4\sqrt{2}\sin^3(x)\sqrt{\cos(2x)}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)+\sin^3(x)\sqrt{-\text{co}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cot[x]^2)^(3/2),x]
```

```
[Out] ((1 - Cot[x]^2)^(3/2)*Sec[2*x]^2*(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Sqrt[-Cos[
2*x]]*Sin[x]^3 + 4*ArcTanh[Cos[x]/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Sin[x]^3 -
 4*Sqrt[2]*Sqrt[Cos[2*x]]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]]*Sin[x]^3 - S
in[4*x]/4))/2
```

fricas [B] time = 1.08, size = 110, normalized size = 2.04

$$\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{\cos(2x)+1}\right)\sin(2x)+\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}(\cos(2x)+1)-5\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{\cos(2x)+1}\right)\sin(2\ldots}{2\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*arctan(sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1)
)*sin(2*x) + sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*(cos(2*x) + 1) - 5*arcta
n(sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1))*sin(2*x))/
sin(2*x)
```

giac [B] time = 0.28, size = 257, normalized size = 4.76

$$\frac{1}{4}\left(5\,\pi\mathrm{sgn}\left(\cos(x)\right)-4\,\sqrt{2}\left(\pi\mathrm{sgn}\left(\cos(x)\right)+2\,\arctan\left(-\frac{\left(\frac{\left(\sqrt{2}\,\sqrt{-2\,\cos(x)^2+1}-\sqrt{2}\right)^2}{\cos(x)^2}-4\right)\cos(x)}{4\left(\sqrt{2}\,\sqrt{-2\,\cos(x)^2+1}-\sqrt{2}\right)}\right)\right)+\frac{4\,\sqrt{2}\left(\frac{\sqrt{2}\,\sqrt{-2\,\cos(}{\cos(}\right.}{\left(\frac{\sqrt{2}\,\sqrt{-2\,\cos(x)^2+1}}{\cos(x)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(5*pi*sgn(cos(x)) - 4*sqrt(2)*(pi*sgn(cos(x)) + 2*arctan(-1/4*((sqrt(2)
*sqrt(-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*c
os(x)^2 + 1) - sqrt(2)))) + 4*sqrt(2)*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqr
t(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2)))/(((sqrt(
2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqrt(-2*cos(
x)^2 + 1) - sqrt(2)))^2 + 8) + 10*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2*cos
(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1)
 - sqrt(2))))*sgn(sin(x))
```

maple [A] time = 0.23, size = 51, normalized size = 0.94

$$\frac{\cot(x)\sqrt{1-\left(\cot^{2}(x)\right)}}{2}+\frac{5\arcsin\left(\cot(x)\right)}{2}+2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-\left(\cot^{2}(x)\right)}\,\cot(x)}{-1+\cot^{2}(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cot(x)^2)^(3/2),x)
```

```
[Out] 1/2*cot(x)*(1-cot(x)^2)^(1/2)+5/2*arcsin(cot(x))+2*2^(1/2)*arctan(2^(1/2)*(
1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cot(x)^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cot(x)^2 + 1)^(3/2), x)
```

mupad [B] time = 0.84, size = 104, normalized size = 1.93

$$\frac{5\,\mathrm{asin}\left(\cot(x)\right)}{2}+\frac{\cot(x)\,\sqrt{1-\cot(x)^2}}{2}-\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(-1+\cot(x)\,1\mathrm{i}\right)\,1\mathrm{i}}{2}-\sqrt{1-\cot(x)^2}\,1\mathrm{i}}{\cot(x)-\mathrm{i}}\right)1\mathrm{i}+\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(1+\cot(x)\,1\mathrm{i}\right)\,1\mathrm{i}}{2}+}{\cot(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cot(x)^2)^(3/2),x)
```

```
[Out] (5*asin(cot(x)))/2 + (cot(x)*(1 - cot(x)^2)^(1/2))/2 - 2^(1/2)*log(((2^(1/2
)*(cot(x)*1i - 1)*1i)/2 - (1 - cot(x)^2)^(1/2)*1i)/(cot(x) - 1i))*1i + 2^(1
/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(cot(x)
+ 1i))*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-\cot^{2}\left(x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - cot(x)**2)**(3/2), x)
```

3.39 $\int \sqrt{1-\cot^2(x)}\,dx$

Optimal. Leaf size=32

$$\sin^{-1}(\cot(x))-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)$$

[Out] arcsin(cot(x))-arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3661, 402, 216, 377, 203}

$$\sin^{-1}(\cot(x))-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Cot[x]^2],x]

[Out] ArcSin[Cot[x]] - Sqrt[2]*ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx, x, \cot(x)\right)\right) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cot(x)\right)\\
&= \sin^{-1}(\cot(x)) - 2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\cot(x)}{\sqrt{1-\cot^2(x)}}\right)\\
&= \sin^{-1}(\cot(x)) - \sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 62, normalized size = 1.94

$$\frac{\sin(x)\sqrt{1-\cot^2(x)}\left(\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)-\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\right)}{\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cot[x]^2],x]
```

```
[Out] (Sqrt[1 - Cot[x]^2]*(-ArcTanh[Cos[x]/Sqrt[Cos[2*x]]] + Sqrt[2]*Log[Sqrt[2]*
Cos[x] + Sqrt[Cos[2*x]]])*Sin[x])/Sqrt[Cos[2*x]]
```

fricas [B] time = 0.41, size = 68, normalized size = 2.12

$$\sqrt{2}\arctan\left(\frac{\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{\cos(2x)+1}\right)-\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{\cos(2x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1)) - arc
tan(sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1))
```

giac [C] time = 0.26, size = 170, normalized size = 5.31

$$-\frac{1}{2}\left(\pi-\sqrt{2}\pi-2\sqrt{2}\arctan\left(-\frac{1}{2}i\sqrt{2}\right)+2\arctan(-i)\right)\text{sgn}(\sin(x))+\frac{1}{2}\left(\pi\text{sgn}(\cos(x))-\sqrt{2}\left(\pi\text{sgn}(\cos(x))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(pi - sqrt(2)*pi - 2*sqrt(2)*arctan(-1/2*I*sqrt(2)) + 2*arctan(-I))*sg
n(sin(x)) + 1/2*(pi*sgn(cos(x)) - sqrt(2)*(pi*sgn(cos(x)) + 2*arctan(-1/4*(
(sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*s
qrt(-2*cos(x)^2 + 1) - sqrt(2)))) + 2*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2
*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2
+ 1) - sqrt(2))))*sgn(sin(x))
```

maple [A] time = 0.23, size = 34, normalized size = 1.06

$$\arcsin(\cot(x)) + \sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{1-\left(\cot^2(x)\right)}\ \cot(x)}{-1+\cot^2(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cot(x)^2)^(1/2),x)
```

```
[Out] arcsin(cot(x))+2^(1/2)*arctan(2^(1/2)*(1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(
x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [B] time = 0.96, size = 88, normalized size = 2.75

$$\mathrm{asin}(\cot(x)) - \frac{\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\,(-1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-\cot(x)^2}\ 1\mathrm{i}}{\cot(x)-\mathrm{i}}\right)1\mathrm{i}}{2} + \frac{\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\,(1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-\cot(x)^2}\ 1\mathrm{i}}{\cot(x)+1\mathrm{i}}\right)1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cot(x)^2)^(1/2),x)
```

```
[Out] asin(cot(x)) - (2^(1/2)*log(((2^(1/2)*(cot(x)*1i - 1)*1i)/2 - (1 - cot(x)^2
)^(1/2)*1i)/(cot(x) - 1i))*1i)/2 + (2^(1/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1
i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(cot(x) + 1i))*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\cot^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cot(x)**2), x)
```

3.40 $\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3661, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Cot[x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]]/Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx, x, \cot(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\cot(x)}{\sqrt{1-\cot^2(x)}}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 1.50

$$-\frac{\sqrt{\cos(2x)}\csc(x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{\sqrt{2-2\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Cot[x]^2],x]
```

```
[Out] -((Sqrt[Cos[2*x]]*Csc[x]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/Sqrt[2 - 2*C
ot[x]^2])
```

fricas [B] time = 0.44, size = 56, normalized size = 2.00

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)}{4\left(\cos(2x)^2+\cos(2x)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(2*sqrt(2)*cos(2*x) + sqrt(2))*sqrt(cos(2*x)
/(cos(2*x) - 1))*sin(2*x)/(cos(2*x)^2 + cos(2*x)))
```

giac [C] time = 0.21, size = 34, normalized size = 1.21

$$-\frac{1}{2}i\sqrt{2}\log\left(i\sqrt{2}+i\right)\operatorname{sgn}(\sin(x))-\frac{\sqrt{2}\arcsin\left(\sqrt{2}\cos(x)\right)}{2\operatorname{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(2)*log(I*sqrt(2) + I)*sgn(sin(x)) - 1/2*sqrt(2)*arcsin(sqrt(2)*
cos(x))/sgn(sin(x))
```

maple [A] time = 0.26, size = 31, normalized size = 1.11

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{1-\left(\cot^2(x)\right)}\cot(x)}{-1+\cot^2(x)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cot(x)^2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*(1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(x))
```

maxima [B] time = 0.51, size = 90, normalized size = 3.21

$$\frac{1}{4}\sqrt{2}\arctan\left(\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)+\sin(2x),\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/
2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x), (cos(4*x)^2 + sin(4*x)^2 + 2
*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + cos(2*x))
```

mupad [B] time = 0.63, size = 85, normalized size = 3.04

$$-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-\cot(x)^2}\,1\mathrm{i}}{\cot(x)-\mathrm{i}}\right)1\mathrm{i}}{4}+\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-\cot(x)^2}\,1\mathrm{i}}{\cot(x)+1\mathrm{i}}\right)1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cot(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(co
t(x) + 1i))*1i)/4 - (2^(1/2)*log(((2^(1/2)*(cot(x)*1i - 1)*1i)/2 - (1 - cot
(x)^2)^(1/2)*1i)/(cot(x) - 1i))*1i)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{1-\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - cot(x)**2), x)
```

3.41 $\int \left(-1+\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}+\frac{5}{2}\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

```
[Out] 5/2*arctanh(cot(x)/(-1+cot(x)^2)^(1/2))-2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)
^2)^(1/2))*2^(1/2)-1/2*cot(x)*(-1+cot(x)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3661, 416, 523, 217, 206, 377}

$$-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}+\frac{5}{2}\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Cot[x]^2)^(3/2),x]
```

```
[Out] (5*ArcTanh[Cot[x]/Sqrt[-1 + Cot[x]^2]])/2 - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Cot[
x])/Sqrt[-1 + Cot[x]^2]] - (Cot[x]*Sqrt[-1 + Cot[x]^2])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(-1+\cot^2(x)\right)^{3/2} dx = -\text{Subst}\left(\int \frac{\left(-1+x^2\right)^{3/2}}{1+x^2} dx, x, \cot(x)\right)$$
$$= -\frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{3-5x^2}{\sqrt{-1+x^2}\left(1+x^2\right)} dx, x, \cot(x)\right)$$
$$= -\frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}} dx, x, \cot(x)\right) - 4\,\text{Subst}\left(\int \frac{}{\sqrt{-1}}\right.$$
$$= -\frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right) - 4\,\text{Subst}\left(\int\right.$$
$$= \frac{5}{2}\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right) - 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right) - \frac{1}{2}\cot(x)\sqrt{-1+\cot^2(}$$

Mathematica [A] time = 0.12, size = 121, normalized size = 1.98

$$\frac{1}{2}\left(\cot^2(x)-1\right)^{3/2}\sec^2(2x)\left(-\frac{1}{4}\sin(4x) - 4\sqrt{2}\sin^3(x)\sqrt{\cos(2x)}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right) + \sin^3(x)\sqrt{-\text{co}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Cot[x]^2)^(3/2),x]
```

```
[Out] ((-1 + Cot[x]^2)^(3/2)*Sec[2*x]^2*(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Sqrt[-Cos
[2*x]]*Sin[x]^3 + 4*ArcTanh[Cos[x]/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Sin[x]^3
- 4*Sqrt[2]*Sqrt[Cos[2*x]]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]]*Sin[x]^3 -
Sin[4*x]/4))/2
```

fricas [B] time = 0.52, size = 170, normalized size = 2.79

$$\frac{4\sqrt{2}\log\left(2\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x) - 2\cos(2x) - 1\right)\sin(2x) - 2\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}(\cos(2x)+1) + 5\log\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{4\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*log(2*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) - 2*cos(2*x) -
 1)*sin(2*x) - 2*sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*(cos(2*x) + 1) + 5*
log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) + cos(2*x) + 1)/(cos(2
*x) + 1))*sin(2*x) - 5*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x)
 - cos(2*x) - 1)/(cos(2*x) + 1))*sin(2*x))/sin(2*x)
```

giac [B] time = 0.66, size = 179, normalized size = 2.93

$$\frac{1}{4}\left(4\sqrt{2}\log\left(\left(\sqrt{2}\cos(x)-\sqrt{2\cos(x)^2-1}\right)^2\right) - \frac{4\sqrt{2}\left(3\left(\sqrt{2}\cos(x)-\sqrt{2\cos(x)^2-1}\right)^2-1\right)}{\left(\sqrt{2}\cos(x)-\sqrt{2\cos(x)^2-1}\right)^4 - 6\left(\sqrt{2}\cos(x)-\sqrt{2\cos(x)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(4*sqrt(2)*log((sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2) - 4*sqrt(2)*(
3*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 - 1)/((sqrt(2)*cos(x) - sqrt(2*
cos(x)^2 - 1))^4 - 6*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 + 1) + 5*log
(abs(2*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 - 4*sqrt(2) - 6)/abs(2*(sq
rt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 + 4*sqrt(2) - 6)))*sgn(sin(x))
```

maple [A] time = 0.22, size = 48, normalized size = 0.79

$$-\frac{\cot(x)\sqrt{-1+\cot^2(x)}}{2}+\frac{5\ln\left(\cot(x)+\sqrt{-1+\cot^2(x)}\right)}{2}-2\,\mathrm{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cot(x)^2)^(3/2),x)
```

```
[Out] -1/2*cot(x)*(-1+cot(x)^2)^(1/2)+5/2*ln(cot(x)+(-1+cot(x)^2)^(1/2))-2*arctan
h(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cot(x)^2-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cot(x)^2 - 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\cot(x)^2-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 - 1)^(3/2),x)
```

```
[Out] int((cot(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cot^2(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)**2)**(3/2),x)
```

```
[Out] Integral((cot(x)**2 - 1)**(3/2), x)
```

3.42 $\int \sqrt{-1+\cot^2(x)}\,dx$

Optimal. Leaf size=42

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

```
[Out] -arctanh(cot(x)/(-1+cot(x)^2)^(1/2))+arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(
1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3661, 402, 217, 206, 377}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Cot[x]^2],x]
```

```
[Out] -ArcTanh[Cot[x]/Sqrt[-1 + Cot[x]^2]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cot[x])/Sqr
t[-1 + Cot[x]^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{-1+\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{1+x^2}\,dx,x,\cot(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)}\,dx,x,\cot(x)\right)-\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,\cot(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)\\
&= -\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 60, normalized size = 1.43

$$
\frac{\sin(x)\sqrt{\cot^2(x)-1}\left(\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)-\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\right)}{\sqrt{\cos(2x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Cot[x]^2],x]
```

```
[Out] (Sqrt[-1 + Cot[x]^2]*(-ArcTanh[Cos[x]/Sqrt[Cos[2*x]]] + Sqrt[2]*Log[Sqrt[2]
*Cos[x] + Sqrt[Cos[2*x]]])*Sin[x])/Sqrt[Cos[2*x]]
```

fricas [B] time = 1.56, size = 123, normalized size = 2.93

$$
\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-2\cos(2x)-1\right)-\frac{1}{2}\log\left(\frac{\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)+\cos(2x)+1}{\cos(2x)+1}\right)+\frac{1}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-2*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) - 2*cos(2*x) - 1
) - 1/2*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) + cos(2*x) + 1
)/(cos(2*x) + 1)) + 1/2*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x
) - cos(2*x) - 1)/(cos(2*x) + 1))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.23, size = 35, normalized size = 0.83

$$
-\ln\left(\cot(x)+\sqrt{-1+\cot^2(x)}\right)+\text{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)\sqrt{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cot(x)^2)^(1/2),x)
```

```
[Out] -ln(cot(x)+(-1+cot(x)^2)^(1/2))+arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))
*2^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [B] time = 0.43, size = 34, normalized size = 0.81

$$\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{\cot(x)^2-1}}\right)-\ln\left(\cot(x)+\sqrt{\cot(x)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 - 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*cot(x))/(cot(x)^2 - 1)^(1/2)) - log(cot(x) + (cot(x)
^2 - 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cot^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(cot(x)**2 - 1), x)
```

3.43 $\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx$

Optimal. Leaf size=26

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3661, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Cot[x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Cot[x])/Sqrt[-1 + Cot[x]^2]]/Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)}\,dx, x, \cot(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.73

$$-\frac{\sqrt{\cos(2x)}\csc(x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{\sqrt{2}\sqrt{\cot^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Cot[x]^2],x]
```

```
[Out] -((Sqrt[Cos[2*x]]*Csc[x]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(Sqrt[2]*Sqr
t[-1 + Cot[x]^2]))
```

fricas [B] time = 0.52, size = 60, normalized size = 2.31

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\left(2\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-8\cos(2x)^2-8\cos(2x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*(2*sqrt(2)*cos(2*x) + sqrt(2))*sqrt(-cos(2*x)/(co
s(2*x) - 1))*sin(2*x) - 8*cos(2*x)^2 - 8*cos(2*x) - 1)
```

giac [B] time = 4.07, size = 45, normalized size = 1.73

$$-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}-1\right)\mathrm{sgn}(\sin(x))+\frac{\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)}{2\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(sqrt(2) - 1)*sgn(sin(x)) + 1/2*sqrt(2)*log(abs(-sqrt(2)*co
s(x) + sqrt(2*cos(x)^2 - 1)))/sgn(sin(x))
```

maple [A] time = 0.25, size = 21, normalized size = 0.81

$$-\frac{\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+cot(x)^2)^(1/2),x)
```

```
[Out] -1/2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

maxima [B] time = 0.50, size = 143, normalized size = 5.50

$$-\frac{1}{8}\sqrt{2}\left(2\,\mathrm{arsinh}(1)+\log\left(\cos(2x)^2+\sin(2x)^2+\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\left(\cos\left(\frac{1}{2}\arctan(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(2*arcsinh(1) + log(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2
+ sin(4*x)^2 + 2*cos(4*x) + 1)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2
+ sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2
```

```
+ 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
+ sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)))))
```

mupad [**B**] time = 0.50, size = 20, normalized size = 0.77

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{\cot(x)^2-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1)^(1/2),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*cot(x))/(cot(x)^2 - 1)^(1/2)))/2
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cot^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(cot(x)**2 - 1), x)
```

3.44 $\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{a+b\cot^2(x)}}{b}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-(a+b*cot(x)^2)^(1/2)
/b
```

Rubi [A] time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 446, 80, 63, 208}

$$-\frac{\sqrt{a+b\cot^2(x)}}{b}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]) - Sqrt[a + b*Cot[x
]^2]/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{\sqrt{a+b\cot^2(x)}}{b}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\\
&= -\frac{\sqrt{a+b\cot^2(x)}}{b}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{\sqrt{a+b\cot^2(x)}}{b}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 52, normalized size = 1.00

$$-\frac{\sqrt{a+b\cot^2(x)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(((b*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]])/Sqrt[a - b] + Sqrt[a + b*C
ot[x]^2])/b)
```

fricas [B] time = 0.66, size = 284, normalized size = 5.46

$$\left[\frac{\sqrt{a-b}\,b\log\left(-2\left(a^2-2\,ab+b^2\right)\cos\left(2\,x\right)^2-2\,a^2+b^2+2\left((a-b)\cos\left(2\,x\right)^2-(2\,a-b)\cos\left(2\,x\right)+a\right)\sqrt{a-b}\,\sqrt{(}\right.}{4\left(ab-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*b*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 + 2
*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*co
s(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) - 4*(a - b)*sqrt(
((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a*b - b^2), -1/2*(sqrt(-a + b)
*b*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(co
s(2*x) - 1)/((a - b)*cos(2*x) - a)) + 2*(a - b)*sqrt(((a - b)*cos(2*x) - a
- b)/(cos(2*x) - 1)))/(a*b - b^2)]
```

giac [B] time = 4.91, size = 127, normalized size = 2.44

$$-\frac{\log\left(\left|2\left(\sqrt{a-b}\,\cos(x)^2-\sqrt{a\cos(x)^4-b\cos(x)^4-2\,a\cos(x)^2+b\cos(x)^2+a}\right)\sqrt{a-b}\,-2\,a+b\right|\right)}{2\,\sqrt{a-b}}+\frac{}{\sqrt{a-b}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*(sqrt(a - b)*cos(x)^2 - sqrt(a*cos(x)^4 - b*cos(x)^4 - 2*a*c
os(x)^2 + b*cos(x)^2 + a))*sqrt(a - b) - 2*a + b))/sqrt(a - b) + 1/(sqrt(a
- b)*cos(x)^2 - sqrt(a*cos(x)^4 - b*cos(x)^4 - 2*a*cos(x)^2 + b*cos(x)^2 +
a) - sqrt(a - b))
```

maple [A] time = 0.21, size = 44, normalized size = 0.85

$$-\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{b}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -(a+b*cot(x)^2)^(1/2)/b+1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(
1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 1.21, size = 44, normalized size = 0.85

$$-\frac{\sqrt{b\cot(x)^2+a}}{b}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] - (a + b*cot(x)^2)^(1/2)/b - atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a
 - b)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**3/sqrt(a + b*cot(x)**2), x)
```

3.45 $\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=64

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(1/2)-arctanh(cot(x)*
b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 483, 217, 206, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[a - b] - ArcTanh[(Sq
rt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = -\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \cot(x)\right) + \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) + \text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

Mathematica [B] time = 0.27, size = 158, normalized size = 2.47

$$\frac{\sin(x)\sqrt{\csc^2(x)((b-a)\cos(2x)+a+b)}\left(\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{-b}\cos(x)}{\sqrt{(a-b)\cos(2x)-a-b}}\right)-\sqrt{-b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a-b}\cos(x)}{\sqrt{(a-b)\cos(2x)-a-b}}\right)\right)}{\sqrt{-b}\sqrt{a-b}\sqrt{(a-b)\cos(2x)-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ((-(Sqrt[-b]*ArcTanh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos
[2*x]]]) + Sqrt[a - b]*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/Sqrt[-a - b + (a -
 b)*Cos[2*x]]])*Sqrt[(a + b + (-a + b)*Cos[2*x])*Csc[x]^2]*Sin[x])/(Sqrt[a
- b]*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]])
```

fricas [B] time = 0.64, size = 588, normalized size = 9.19

$$\left[-\frac{\sqrt{-a+b}\,b\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+b\right)-(a-b)\sqrt{b}\log\left(\frac{(a-2b)\cos(2x)+2\right.}{}\right.}{2\left(ab-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a + b)*b*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*co
s(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b) - (a - b)*sqrt(b)*log(((a - 2
*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*si
n(2*x) - a - 2*b)/(cos(2*x) - 1)))/(a*b - b^2), 1/2*(2*(a - b)*sqrt(-b)*arc
tan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*co
```

```
s(2*x) + b)) - sqrt(-a + b)*b*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a
 - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b))/(a*b - b^2), 1/2*(2*
sqrt(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x)
 - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b)) + (a - b)*sqrt(b)*log(((a - 2*b
)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(
2*x) - a - 2*b)/(cos(2*x) - 1)))/(a*b - b^2), (sqrt(a - b)*b*arctan(-sqrt(a
 - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos
(2*x) + a - b)) + (a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) -
 a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b)))/(a*b - b^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(t_nostep))]Discontinuities at zeroes of sin(t_nostep) were not checkedWarn
ing, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(t_nostep)]Error: Bad Argument Type
```

maple [A] time = 0.19, size = 80, normalized size = 1.25

$$-\frac{\ln\left(\cot(x)\sqrt{b}+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{\sqrt{b}}+\frac{\sqrt{b^4(a-b)}\ \arctan\left(\frac{(a-b)b^2\cot(x)}{\sqrt{b^4(a-b)}\ \sqrt{a+b\left(\cot^2(x)\right)}}\right)}{b^2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -ln(cot(x)*b^(1/2)+(a+b*cot(x)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b
)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)^2}{\sqrt{b\cot(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)^2/sqrt(b*cot(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cot(x)}^2}{\sqrt{b\,{\cot(x)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(cot(x)^2/(a + b*cot(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+b*cot(x)**2)**(1/2),x)
[Out] Integral(cot(x)**2/sqrt(a + b*cot(x)**2), x)
```

3.46 $\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3670, 444, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx = -\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Cot[x]^2],x]
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]
```

fricas [B] time = 0.52, size = 127, normalized size = 3.85

$$\left[\frac{\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1)-(a-b)\cos(2x)+a\right)}{2\sqrt{a-b}}, \frac{\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{a-b}\right)}{a-b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(
2*x) - 1) - (a - b)*cos(2*x) + a)/sqrt(a - b), sqrt(-a + b)*arctan(-sqrt(-a
 + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a - b))/(a - b)]
```

giac [B] time = 0.56, size = 70, normalized size = 2.12

$$\frac{\log\left(\left|2\left(\sqrt{a-b}\cos(x)^2-\sqrt{a\cos(x)^4-b\cos(x)^4-2a\cos(x)^2+b\cos(x)^2+a}\right)\sqrt{a-b}-2a+b\right|\right)}{2\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(2*(sqrt(a - b)*cos(x)^2 - sqrt(a*cos(x)^4 - b*cos(x)^4 - 2*a*co
s(x)^2 + b*cos(x)^2 + a))*sqrt(a - b) - 2*a + b))/sqrt(a - b)
```

maple [A] time = 0.15, size = 29, normalized size = 0.88

$$-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 0.96, size = 27, normalized size = 0.82

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*cot(x)**2), x)
```

3.47 $\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(1/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))/(a-b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
```

```
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

fricas [A] time = 0.70, size = 419, normalized size = 6.98

$$\left[\frac{(a-b)\sqrt{a}\,\log\left(2\,a\tan(x)^2+2\,\sqrt{a}\,\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\,\tan(x)^2+b\right)+\sqrt{a-b}\,a\log\left(\frac{(2\,a-b)\tan(x)^2-2\,\sqrt{a-b}\,\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\,\tan(x)^2+b}{\tan(x)^2+1}\right.}{2\left(a^2-ab\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a - b)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/ta
n(x)^2)*tan(x)^2 + b) + sqrt(a - b)*a*log(((2*a - b)*tan(x)^2 - 2*sqrt(a -
b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a^2 - a*
b), -1/2*(2*a*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x
)^2)/(a - b)) - (a - b)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)
^2 + b)/tan(x)^2)*tan(x)^2 + b))/(a^2 - a*b), -1/2*(2*sqrt(-a)*(a - b)*arct
an(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) - sqrt(a - b)*a*log(((2*a -
b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(
```

```
tan(x)^2 + 1)))/(a^2 - a*b), -(sqrt(-a)*(a - b)*arctan(sqrt(-a)*sqrt((a*tan
(x)^2 + b)/tan(x)^2)/a) + a*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x
)^2 + b)/tan(x)^2)/(a - b)))/(a^2 - a*b)]
```

giac [B] time = 0.38, size = 140, normalized size = 2.33

$$-\frac{\arctan\left(-\frac{\sqrt{a-b}\cos(x)^2-\sqrt{a\cos(x)^4-b\cos(x)^4-2\,a\cos(x)^2+b\cos(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\log\left(\left|-2\left(\sqrt{a-b}\cos(x)^2-\sqrt{a\cos(x)^4-b\cos(x\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(-(sqrt(a - b)*cos(x)^2 - sqrt(a*cos(x)^4 - b*cos(x)^4 - 2*a*cos(x)^
2 + b*cos(x)^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*log(abs(-2*(sqrt(a - b)*cos(x
)^2 - sqrt(a*cos(x)^4 - b*cos(x)^4 - 2*a*cos(x)^2 + b*cos(x)^2 + a))*(a - b
) + (2*a - b)*sqrt(a - b)))/sqrt(a - b)
```

maple [C] time = 0.75, size = 376, normalized size = 6.27

$$\frac{2\sqrt{2}\sqrt{\frac{\cos(x)\sqrt{a}\sqrt{a-b}-\sqrt{a}\sqrt{a-b}-a\cos(x)+b\cos(x)+a}{(\cos(x)+1)b}}\sqrt{-\frac{2\left(\cos(x)\sqrt{a}\sqrt{a-b}-\sqrt{a}\sqrt{a-b}+a\cos(x)-b\cos(x)-a\right)}{(\cos(x)+1)b}}\left(\mathrm{EllipticPi}\left(\frac{(-1+\mathrm{co}}{}\right.\right.}{\sqrt{\frac{a\left(\cos^2(x)\right)-b\left(\cos^2\right.}{-1+\cos^2(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] 2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x
)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2
)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*(EllipticPi((-1+cos(x))*((2*a^(1
/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-
(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1
/2))-EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),
-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)
/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)))*sin(x)/((a*cos(x)^2-b*cos(x)^2-a
)/(-1+cos(x)^2))^(1/2)/(-1+cos(x))/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{b\cot(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(b*cot(x)^2 + a), x)
```

mupad [B] time = 0.51, size = 93, normalized size = 1.55

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan(x)^2}}}{\sqrt{a-b}}+\frac{2\sqrt{a-b}\sqrt{a+\frac{b}{\tan(x)^2}}}{b}-\frac{2\,a\sqrt{a+\frac{b}{\tan(x)^2}}}{b\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan(x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b/tan(x)^2)^(1/2)/(a - b)^(1/2) + (2*(a - b)^(1/2)*(a + b/tan(x)
^2)^(1/2))/b - (2*a*(a + b/tan(x)^2)^(1/2))/(b*(a - b)^(1/2)))/(a - b)^(1/2
) + atanh((a + b/tan(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a + b\cot^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cot(x)**2), x)
```

3.48 $\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}+\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}$$

[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(1/2)+(a+b*cot(x)^2)^(1/2)*tan(x)/a

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 480, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}+\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]^2/Sqrt[a + b*Cot[x]^2],x]

[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[a - b] + (Sqrt[a + b*Cot[x]^2]*Tan[x])/a

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
```

```
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)$$
$$= \frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a} + \frac{\text{Subst}\left(\int \frac{a}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)}{a}$$
$$= \frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a} + \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)$$
$$= \frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a} + \text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a}$$

Mathematica [C] time = 1.57, size = 134, normalized size = 2.48

$$\frac{\sin^2(x)\tan(x)\left(\frac{b\cot^2(x)}{a}+1\right)\left(\frac{3a\left(a+2b\cot^2(x)\right)\sin^{-1}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)}{\sqrt{\frac{(a-b)\sin^2(x)\cos^2(x)\left(a+b\cot^2(x)\right)}{a^2}}} + 4(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(a-b)\cos\ldots}{a}\right.\right.}{3a^2\sqrt{a+b\cot^2(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ((1 + (b*Cot[x]^2)/a)*Sin[x]^2*(4*(a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Hyperge
ometric2F1[2, 2, 5/2, ((a - b)*Cos[x]^2)/a] + (3*a*ArcSin[Sqrt[((a - b)*Cos
[x]^2)/a]]*(a + 2*b*Cot[x]^2))/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[
x]^2)/a^2])*Tan[x])/(3*a^2*Sqrt[a + b*Cot[x]^2])
```

fricas [A] time = 1.11, size = 229, normalized size = 4.24

$$\left[-\frac{a\sqrt{-a+b}\,\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2+4\left(a\tan(x)^3-(a-2b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{\tan(x)^4+2\tan(x)^2+1}\right)-4(a-b)\sqrt{\frac{a\tan(x\ldots}{\tan(\ldots}}}{4\left(a^2-ab\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2
 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*t
an(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) - 4*(a - b)*sqrt((a*ta
n(x)^2 + b)/tan(x)^2)*tan(x))/(a^2 - a*b), 1/2*(sqrt(a - b)*a*arctan(2*sqrt
```

```
(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + 2*
(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x))/(a^2 - a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(t_nostep))]Discontinuities at zeroes of sin(t_nostep) were not checkedWarn
ing, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(t_nostep)]Error: Bad Argument Type
```

maple [B] time = 0.82, size = 328, normalized size = 6.07

$$-\frac{\sin(x)\left(\left(\cos^2(x)\right)\sqrt{-\frac{a\left(\cos^2(x)\right)-b\left(\cos^2(x)\right)-a}{(\cos(x)+1)^2}}\ \ln\left(4\cos(x)\sqrt{-a+b}\ \sqrt{-\frac{a\left(\cos^2(x)\right)-b\left(\cos^2(x)\right)-a}{(\cos(x)+1)^2}}-4a\cos(x)+4b\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*cot(x)^2)^(1/2),x)
```

```
[Out] -sin(x)*(cos(x)^2*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(
x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+
4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*
a-cos(x)^2*(-a+b)^(1/2)*a+cos(x)^2*(-a+b)^(1/2)*b+cos(x)*(-(a*cos(x)^2-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(
x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x
)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a+(-a+b)^(1/2)*a)/cos(x)/((a*cos(x)^
2-b*cos(x)^2-a)/(-1+cos(x)^2))^(1/2)/(-1+cos(x)^2)/a/(-a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)^2}{\sqrt{b\cot(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^2/sqrt(b*cot(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\tan(x)}^2}{\sqrt{b\,{\cot(x)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2/(a + b*cot(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a + b*cot(x)**2), x)
```

3.49 $\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{a}{b(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

```
[Out] -arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)+a/(a-b)/b/(a+b*cot(x
)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 446, 78, 63, 208}

$$\frac{a}{b(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(3/2)) + a/((a - b)*b*S
qrt[a + b*Cot[x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx,x,\cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{3/2}}\,dx,x,\cot^2(x)\right)\right)\\
&= \frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,\cot^2(x)\right)}{2(a-b)}\\
&= \frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cot^2(x)}\right)}{(a-b)b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 59, normalized size = 1.00

$$\frac{a}{b(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(3/2)) + a/((a - b)*b*S
qrt[a + b*Cot[x]^2])
```

fricas [B] time = 0.45, size = 385, normalized size = 6.53

$$\left[-\frac{\left(ab+b^2-\left(ab-b^2\right)\cos\left(2x\right)\right)\sqrt{a-b}\,\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\left(\cos\left(2x\right)-1\right)-(a-b)\cos\left(2x\right)+a\right)-2}{2\left(a^3b-a^2b^2-ab^3+b^4-\left(a^3b-3a^2b^2+3ab^3-b^4\right)\cos\left(2x\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*b + b^2 - (a*b - b^2)*cos(2*x))*sqrt(a - b)*log(-sqrt(a - b)*sqrt
(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*
x) + a) - 2*(a^2 - a*b - (a^2 - a*b)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a -
```

```
 b)/(cos(2*x) - 1)))/(a^3*b - a^2*b^2 - a*b^3 + b^4 - (a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*cos(2*x)), -((a*b + b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)
*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a -
b)) - (a^2 - a*b - (a^2 - a*b)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(c
os(2*x) - 1)))/(a^3*b - a^2*b^2 - a*b^3 + b^4 - (a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4)*cos(2*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[%%%{2,[1,2]%%%}+%%%{-2,[0,3]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,
[0,1]%%%}]%%},[2]%%%}+%%%{%%%{4,[2,2]%%%}+%%%{-4,[1,3]%%%},[1]%%%}+%%%{%%{[
%%%{2,[2,2]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[0]%%%} / %%%{
%%%{1,[2,0]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,2]%%%},[2]%%%}+%%%{%%{[%%%{2,[2,0
]%%%}+%%%{-2,[1,1]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1]%%%}
+%%%{%%%{1,[3,0]%%%}+%%%{-1,[2,1]%%%},[0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.17, size = 68, normalized size = 1.15

$$\frac{1}{b\sqrt{a+b\left(\cot^2(x)\right)}}+\frac{1}{(a-b)\sqrt{a+b\left(\cot^2(x)\right)}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] 1/b/(a+b*cot(x)^2)^(1/2)+1/(a-b)/(a+b*cot(x)^2)^(1/2)+1/(a-b)/(-a+b)^(1/2)*
arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 1.92, size = 52, normalized size = 0.88

$$\frac{a}{\left(a\,b-b^2\right)\,\sqrt{b\,{\mathrm{cot}\left(x\right)}^2+a}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,{\mathrm{cot}\left(x\right)}^2+a}}{\sqrt{a-b}}\right)}{{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] a/((a*b - b^2)*(a + b*cot(x)^2)^(1/2)) - atanh((a + b*cot(x)^2)^(1/2)/(a -
b)^(1/2))/(a - b)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(x)}{\left(a + b\cot^2(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)**3/(a + b*cot(x)**2)**(3/2), x)
```

3.50 $\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(3/2)-cot(x)/(a-b)/(a
+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3670, 471, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(3/2) - Cot[x]/((
a - b)*Sqrt[a + b*Cot[x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)$$

$$= -\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)}{a-b}$$

$$= -\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{a-b}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}} - \frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

Mathematica [B] time = 0.73, size = 137, normalized size = 2.32

$$\frac{(b-a)\cot(x)\sqrt{\frac{b\cot^2(x)}{a}+1}+\frac{1}{2}\csc(x)\sec(x)((a-b)\cos(2x)-a-b)\sqrt{-\frac{(a-b)\cot^2(x)}{a}}\tanh^{-1}\left(\frac{\sqrt{-\frac{(a-b)\cot^2(x)}{a}}}{\sqrt{\frac{b\cot^2(x)}{a}+1}}\right)}{(a-b)^2\sqrt{a+b\cot^2(x)}\sqrt{\frac{b\cot^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ((-a + b)*Cot[x]*Sqrt[1 + (b*Cot[x]^2)/a] + (ArcTanh[Sqrt[-(((a - b)*Cot[x]
^2)/a)]/Sqrt[1 + (b*Cot[x]^2)/a]]*(-a - b + (a - b)*Cos[2*x])*Sqrt[-(((a -
b)*Cot[x]^2)/a)]*Csc[x]*Sec[x])/2)/((a - b)^2*Sqrt[a + b*Cot[x]^2]*Sqrt[1 +
 (b*Cot[x]^2)/a])
```

fricas [B] time = 0.74, size = 388, normalized size = 6.58

$$\left[-\frac{((a-b)\cos(2x)-a-b)\sqrt{-a+b}\,\log\left(-2\left(a^2-2ab+b^2\right)\cos(2x)^2-2((a-b)\cos(2x)-b)\sqrt{-a+b}\sqrt{\frac{(a-b)\,c}{\cos}}\right.}{4\left(a^3-a^2b-ab^2+b^3-\left(a^3-3a^2b+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - b)*cos(2*x) - a - b)*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*c
os(2*x)^2 - 2*((a - b)*cos(2*x) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*x) -
a - b)/(cos(2*x) - 1))*sin(2*x) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*x)) + 4
*(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^3 - a
^2*b - a*b^2 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x)), -1/2*(((a -
 b)*cos(2*x) - a - b)*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) - b)) + 2*(a - b)*sqr
t(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^3 - a^2*b - a*b^2
 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x))]
```

giac [B] time = 1.10, size = 259, normalized size = 4.39

$$\frac{\frac{\left(a^{3}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)-2a^{2}b\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)+ab^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\right)\tan\left(\frac{1}{2}x\right)^{2}}{a^{4}-3a^{3}b+3a^{2}b^{2}-ab^{3}}-\frac{a^{3}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)-2a^{2}b\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)+ab^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}{a^{4}-3a^{3}b+3a^{2}b^{2}-ab^{3}}}{\sqrt{b\tan\left(\frac{1}{2}x\right)^{4}+4a\tan\left(\frac{1}{2}x\right)^{2}-2b\tan\left(\frac{1}{2}x\right)^{2}+b}}+\frac{\sqrt{b}\operatorname{sgn}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*sgn(tan(1/2*x)) - 2*a^2*b*sgn(tan(1/2*x)) + a*b^2*sgn(tan(1/2*x)))*ta
n(1/2*x)^2/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3) - (a^3*sgn(tan(1/2*x)) - 2*a
^2*b*sgn(tan(1/2*x)) + a*b^2*sgn(tan(1/2*x)))/(a^4 - 3*a^3*b + 3*a^2*b^2 -
a*b^3))/sqrt(b*tan(1/2*x)^4 + 4*a*tan(1/2*x)^2 - 2*b*tan(1/2*x)^2 + b) + sq
rt(b)*sgn(tan(1/2*x))/(a*b - b^2) - 2*arctan(-1/2*(sqrt(b)*tan(1/2*x)^2 - s
qrt(b*tan(1/2*x)^4 + 4*a*tan(1/2*x)^2 - 2*b*tan(1/2*x)^2 + b) + sqrt(b))/sq
rt(a - b))/((a*sgn(tan(1/2*x)) - b*sgn(tan(1/2*x)))*sqrt(a - b))
```

maple [A] time = 0.18, size = 99, normalized size = 1.68

$$-\frac{\cot(x)}{a\sqrt{a+b\left(\cot^{2}(x)\right)}}-\frac{b\cot(x)}{(a-b)\,a\sqrt{a+b\left(\cot^{2}(x)\right)}}+\frac{\sqrt{b^{4}(a-b)}\;\arctan\left(\frac{(a-b)b^{2}\cot(x)}{\sqrt{b^{4}(a-b)}\;\sqrt{a+b\left(\cot^{2}(x)\right)}}\right)}{(a-b)^{2}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] -cot(x)/a/(a+b*cot(x)^2)^(1/2)-b/(a-b)*cot(x)/a/(a+b*cot(x)^2)^(1/2)+1/(a-b
)^2*(b^4*(a-b))^(1/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)
^(1/2)*cot(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)^{2}}{\left(b\cot(x)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2/(a + b*cot(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+b*cot(x)**2)**(3/2),x)
[Out] Integral(cot(x)**2/(a + b*cot(x)**2)**(3/2), x)
```

3.51 $\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)-1/(a-b)/(a+b*cot(x)^2
)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(3/2) - 1/((a - b)*Sqrt[a
 + b*Cot[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)}{2(a-b)}\\
&= -\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{(a-b)b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} - \frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 44, normalized size = 0.80

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\cot^2(x)+a}{a-b}\right)}{(b-a)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Cot[x]^2)/(a - b)]/((-a + b)*Sqrt[a
+ b*Cot[x]^2])
```

fricas [B] time = 0.47, size = 344, normalized size = 6.25

$$\left[\frac{((a-b)\cos(2x)-a-b)\sqrt{a-b}\,\log\left(\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,(\cos(2x)-1)-(a-b)\cos(2x)+a\right)+2\,((a-b)}{2\left(a^3-a^2b-ab^2+b^3-\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\cos(2x)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a - b)*cos(2*x) - a - b)*sqrt(a - b)*log(sqrt(a - b)*sqrt(((a - b)*
cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) +
2*((a - b)*cos(2*x) - a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)
))/(a^3 - a^2*b - a*b^2 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x)),
```

```
-(((a - b)*cos(2*x) - a - b)*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b
)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a - b)) - ((a - b)*cos(2*x) - a + b)*s
qrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^3 - a^2*b - a*b^2 + b^3
- (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[%%%{2,[1,2]%%%}+%%%{-2,[0,3]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,
[0,1]%%%}]%%},[2]%%%}+%%%{%%%{4,[2,2]%%%}+%%%{-4,[1,3]%%%},[1]%%%}+%%%{%%{[
%%%{2,[2,2]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[0]%%%} / %%%{
%%%{1,[2,0]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,2]%%%},[2]%%%}+%%%{%%{[%%%{2,[2,0
]%%%}+%%%{-2,[1,1]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1]%%%}
+%%%{%%%{1,[3,0]%%%}+%%%{-1,[2,1]%%%},[0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.14, size = 56, normalized size = 1.02

$$-\frac{1}{(a-b)\sqrt{a+b\left(\cot^2(x)\right)}}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] -1/(a-b)/(a+b*cot(x)^2)^(1/2)-1/(a-b)/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1
/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 1.78, size = 47, normalized size = 0.85

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{1}{(a-b)\sqrt{b\cot(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a - b)^(3/2) - 1/((a - b)*(a +
 b*cot(x)^2)^(1/2))
```

sympy [A] time = 10.37, size = 48, normalized size = 0.87

$$-\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] -1/((a - b)*sqrt(a + b*cot(x)**2)) - atan(sqrt(a + b*cot(x)**2)/sqrt(-a + b
))/(sqrt(-a + b)*(a - b))
```

3.52 $\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(3/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))/(a-b)^(3/2)+b/a/(a-b)/(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3670, 446, 85, 156, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/a^(3/2) - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/(a - b)^(3/2) + b/(a*(a - b)*Sqrt[a + b*Cot[x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)\right)$$

$$= \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)}{2a(a-b)}$$

$$= \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x,\right.}{2(a-b)}$$

$$= \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{ab} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{(a-}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

Mathematica [C] time = 0.06, size = 75, normalized size = 0.89

$$\frac{a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\cot^2(x)+a}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\cot^2(x)}{a}+1\right)}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Cot[x]^2)/(a - b)] + (-a + b)*Hyp
ergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cot[x]^2)/a])/(a*(a - b)*Sqrt[a + b*Cot
[x]^2])
```

fricas [B] time = 1.01, size = 863, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b - a*b^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + (a^2*b -
 2*a*b^2 + b^3 + (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)^2
 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b) - (a^3*tan(x)^2
+ a^2*b)*sqrt(a - b)*log(((2*a - b)*tan(x)^2 + 2*sqrt(a - b)*sqrt((a*tan(x)
^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a^4*b - 2*a^3*b^2 + a^2*b
^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), 1/2*(2*(a^2*b - a*b^2)*sqrt((a*ta
n(x)^2 + b)/tan(x)^2)*tan(x)^2 - 2*(a^3*tan(x)^2 + a^2*b)*sqrt(-a + b)*arct
an(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) + (a^2*b - 2*a*b^
2 + b^3 + (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)^2 + 2*sq
rt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b))/(a^4*b - 2*a^3*b^2 + a
^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), 1/2*(2*(a^2*b - a*b^2)*sqrt((
a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 - 2*(a^2*b - 2*a*b^2 + b^3 + (a^3 - 2*a^
2*b + a*b^2)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x
)^2)/a) - (a^3*tan(x)^2 + a^2*b)*sqrt(a - b)*log(((2*a - b)*tan(x)^2 + 2*sq
rt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a
^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), ((a^2*b -
 a*b^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 - (a^2*b - 2*a*b^2 + b^3 +
 (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^
2 + b)/tan(x)^2)/a) - (a^3*tan(x)^2 + a^2*b)*sqrt(-a + b)*arctan(-sqrt(-a +
 b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)))/(a^4*b - 2*a^3*b^2 + a^2*b^3
+ (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, integra
tion of abs or sign assumes constant sign by intervals (correct if the argu
ment is real):Check [abs(t_nostep)]Error: Bad Argument Type
```

maple [C] time = 0.90, size = 962, normalized size = 11.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] -(a*cos(x)^2-b*cos(x)^2-a)*(2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a
-b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b
)^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*Ellipt
icF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),((8*a^(3/2)*
(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))*b*sin(x)+2
*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)
+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)
+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2
)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2
*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2
))*a*sin(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-b)^(1/2)-a*co
```

```
s(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2
)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x
))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2)-
2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2
*a+b)/b)^(1/2))*b*sin(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a^(1/2)*(a-
b)^(1/2)-a*cos(x)+b*cos(x)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)
^(1/2)-a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a)/(cos(x)+1)/b)^(1/2)*Ellipti
cPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2
)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)
*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a*sin(x)+((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1
/2)*b*cos(x)-((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)*b)/(-1+cos(x))/sin(x)^
2/((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))^(3/2)/((2*a^(1/2)*(a-b)^(1/2)-2
*a+b)/b)^(1/2)/(a-b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(b\cot(x)^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*cot(x)^2 + a)^(3/2), x)
```

mupad [B] time = 0.48, size = 1451, normalized size = 17.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh((2*a^2*b^8*(a + b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7
 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) - (12*a^3*b^7*(a + b/
tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b
^5 + 24*a^5*b^4 - 6*a^6*b^3)) + (30*a^4*b^6*(a + b/tan(x)^2)^(1/2))/((a^3)^
(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*
b^3)) - (38*a^5*b^5*(a + b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*
b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) + (24*a^6*b^4*(a +
 b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^
4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) - (6*a^7*b^3*(a + b/tan(x)^2)^(1/2))/((a^3
)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^
6*b^3)))/(a^3)^(1/2) - (atan(((((a - b)^3)^(1/2)*(((a + b/tan(x)^2)^(1/2)*(
2*a^3*b^7 - 10*a^4*b^6 + 22*a^5*b^5 - 26*a^6*b^4 + 16*a^7*b^3 - 4*a^8*b^2))
/2 + (((a - b)^3)^(1/2)*(12*a^5*b^7 - 2*a^4*b^8 - 28*a^6*b^6 + 32*a^7*b^5 -
 18*a^8*b^4 + 4*a^9*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*
b^8 - 56*a^6*b^7 + 160*a^7*b^6 - 240*a^8*b^5 + 200*a^9*b^4 - 88*a^10*b^3 +
16*a^11*b^2))/(4*(a - b)^3)))/(2*(a - b)^3))*1i)/(a - b)^3 + (((a - b)^3)^(
1/2)*(((a + b/tan(x)^2)^(1/2)*(2*a^3*b^7 - 10*a^4*b^6 + 22*a^5*b^5 - 26*a^6
*b^4 + 16*a^7*b^3 - 4*a^8*b^2))/2 + (((a - b)^3)^(1/2)*(2*a^4*b^8 - 12*a^5*
b^7 + 28*a^6*b^6 - 32*a^7*b^5 + 18*a^8*b^4 - 4*a^9*b^3 + ((a + b/tan(x)^2)^
(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8 - 56*a^6*b^7 + 160*a^7*b^6 - 240*a^8*b^5
 + 200*a^9*b^4 - 88*a^10*b^3 + 16*a^11*b^2))/(4*(a - b)^3)))/(2*(a - b)^3))
*1i)/(a - b)^3)/(2*a^3*b^6 - 6*a^4*b^5 + 6*a^5*b^4 - 2*a^6*b^3 - (((a - b)^
3)^(1/2)*(((a + b/tan(x)^2)^(1/2)*(2*a^3*b^7 - 10*a^4*b^6 + 22*a^5*b^5 - 26
*a^6*b^4 + 16*a^7*b^3 - 4*a^8*b^2))/2 + (((a - b)^3)^(1/2)*(12*a^5*b^7 - 2*
a^4*b^8 - 28*a^6*b^6 + 32*a^7*b^5 - 18*a^8*b^4 + 4*a^9*b^3 + ((a + b/tan(x)
^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8 - 56*a^6*b^7 + 160*a^7*b^6 - 240*a^8
*b^5 + 200*a^9*b^4 - 88*a^10*b^3 + 16*a^11*b^2))/(4*(a - b)^3)))/(2*(a - b)
^3)))/(a - b)^3 + (((a - b)^3)^(1/2)*(((a + b/tan(x)^2)^(1/2)*(2*a^3*b^7 -
```

```
10*a^4*b^6 + 22*a^5*b^5 - 26*a^6*b^4 + 16*a^7*b^3 - 4*a^8*b^2))/2 + (((a -
b)^3)^(1/2)*(2*a^4*b^8 - 12*a^5*b^7 + 28*a^6*b^6 - 32*a^7*b^5 + 18*a^8*b^4
- 4*a^9*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8 - 56*a^6
*b^7 + 160*a^7*b^6 - 240*a^8*b^5 + 200*a^9*b^4 - 88*a^10*b^3 + 16*a^11*b^2)
)/(4*(a - b)^3)))/(2*(a - b)^3)))/(a - b)^3))*((a - b)^3)^(1/2)*1i)/(a - b)
^3 - b/((a*b - a^2)*(a + b/tan(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral(tan(x)/(a + b*cot(x)**2)**(3/2), x)
```

3.53 $\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{(a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a^2(a-b)}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}+\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(3/2)+b*tan(x)/a/(a-b
)/(a+b*cot(x)^2)^(1/2)+(a-2*b)*(a+b*cot(x)^2)^(1/2)*tan(x)/a^2/(a-b)
```

Rubi [A] time = 0.15, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 472, 583, 12, 377, 203}

$$\frac{(a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a^2(a-b)}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}+\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(3/2) + (b*Tan[x]
)/(a*(a - b)*Sqrt[a + b*Cot[x]^2]) + ((a - 2*b)*Sqrt[a + b*Cot[x]^2]*Tan[x]
)/(a^2*(a - b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx,x,\cot(x)\right)\\
&= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\text{Subst}\left(\int \frac{a-2b-2bx^2}{x^2\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)}{a(a-b)}\\
&= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}+\frac{(a-2b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{a^2(a-b)}+\frac{\text{Subst}\left(\int \frac{a^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,d\right.}{a^2(a-b)}\\
&= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}+\frac{(a-2b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{a^2(a-b)}+\frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,d\right.}{a-b}\\
&= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}+\frac{(a-2b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{a^2(a-b)}+\frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x\right.}{a-b}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}+\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}+\frac{(a-2b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{a^2(a-b)}
\end{aligned}$$

Mathematica [C] time = 6.90, size = 674, normalized size = 7.33

$$\sin^2(x)\tan(x)\left(\frac{8b^2(a-b)\cos^2(x)\cot^4(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{15a^3}+\frac{16b(a-b)\cos^2(x)\cot^2(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{15a^2}+\frac{8(a-b)\cos^2(x\cdots}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (Sin[x]^2*((12*b*Csc[x]^2)/(a - b) + (8*b^2*Cot[x]^2*Csc[x]^2)/(a*(a - b))
+ (16*(a - b)*Cos[x]^2*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Cos[x]^2)/a])/
(15*a) + (8*(a - b)*b*Cos[x]^2*Cot[x]^2*Hypergeometric2F1[2, 2, 7/2, ((a -
b)*Cos[x]^2)/a])/(3*a^2) + (8*(a - b)*b^2*Cos[x]^2*Cot[x]^4*Hypergeometric2
```

```
F1[2, 2, 7/2, ((a - b)*Cos[x]^2)/a])/(5*a^3) + (8*(a - b)*Cos[x]^2*Hypergeo
metricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Cos[x]^2)/a])/(15*a) + (16*(a - b)*
b*Cos[x]^2*Cot[x]^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Cos[x]^
2)/a])/(15*a^2) + (8*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricPFQ[{2, 2,
 2}, {1, 7/2}, ((a - b)*Cos[x]^2)/a])/(15*a^3) + (3*a*Sec[x]^2)/(a - b) - (
3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]])/((((a - b)*Cos[x]^2)/a)^(3/2)*Sqrt[((
a + b*Cot[x]^2)*Sin[x]^2)/a]) - (12*b*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Co
t[x]^2)/(a*(((a - b)*Cos[x]^2)/a)^(3/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]
) - (8*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^4)/(a^2*(((a - b)*Cos[
x]^2)/a)^(3/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) + (3*ArcSin[Sqrt[((a -
b)*Cos[x]^2)/a]])/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2] +
(12*b*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^2)/(a*Sqrt[((a - b)*Cos[x]^
2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) + (8*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/
a]]*Cot[x]^4)/(a^2*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]))
*Tan[x])/(a*Sqrt[a + b*Cot[x]^2])
```

fricas [B] time = 0.90, size = 393, normalized size = 4.27

$$\left[\frac{\left(a^3\tan(x)^2+a^2b\right)\sqrt{-a+b}\,\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2-4\left(a\tan(x)^3-(a-2b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{\tan(x)^4+2\tan(x)^2+1}\right)+4}{4\left(a^4b-2a^3b^2+a^2b^3+\left(a^5-2a^4b+a^3b^2\right)\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^3*tan(x)^2 + a^2*b)*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4
*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 - 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sq
rt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) +
4*((a^3 - 2*a^2*b + a*b^2)*tan(x)^3 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(x))*sqr
t((a*tan(x)^2 + b)/tan(x)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b
 + a^3*b^2)*tan(x)^2), 1/2*((a^3*tan(x)^2 + a^2*b)*sqrt(a - b)*arctan(2*sqr
t(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + 2
*((a^3 - 2*a^2*b + a*b^2)*tan(x)^3 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(x))*sqrt
((a*tan(x)^2 + b)/tan(x)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b
+ a^3*b^2)*tan(x)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(sin
(t_nostep))]Discontinuities at zeroes of sin(t_nostep) were not checkedWarn
ing, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(t_nostep)]Evaluation time: 0.48Error:
Bad Argument Type
```

maple [B] time = 0.96, size = 421, normalized size = 4.58

$$\frac{(-1+\cos(x))^2(\cos(x)+1)^2\left(a\left(\cos^2(x)\right)-b\left(\cos^2(x)\right)-a\right)\left(-\left(\cos^2(x)\right)\sqrt{-\frac{a\left(\cos^2(x)\right)-b\left(\cos^2(x)\right)-a}{(\cos(x)+1)^2}}\ \ln\left(4\cos(x)\sqrt{-a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*cot(x)^2)^(3/2),x)
```

```
[Out] (-1+cos(x))^2*(cos(x)+1)^2*(a*cos(x)^2-b*cos(x)^2-a)*(-cos(x)^2*(-(a*cos(x)
^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2
-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(
a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^2+cos(x)^2*(-a+b)^(1/2)*a^2
-2*cos(x)^2*(-a+b)^(1/2)*a*b+2*cos(x)^2*(-a+b)^(1/2)*b^2-cos(x)*(-(a*cos(x)
^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2
-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(
a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^2-(-a+b)^(1/2)*a^2+(-a+b)^(
1/2)*a*b)*b/cos(x)/((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))^(3/2)/sin(x)^7
/(-a+b)^(1/2)/((a*(a-b))^(1/2)+a-b)/((a*(a-b))^(1/2)-a+b)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)^2}{\left(b\cot(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^2/(b*cot(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^2}{\left(b\cot(x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(tan(x)^2/(a + b*cot(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral(tan(x)**2/(a + b*cot(x)**2)**(3/2), x)
```

3.54 $\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{a}{3b(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

```
[Out] -arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)+1/3*a/(a-b)/b/(a+b*c
ot(x)^2)^(3/2)+1/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 78, 51, 63, 208}

$$\frac{a}{3b(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(5/2)) + a/(3*(a - b)*b
*(a + b*Cot[x]^2)^(3/2)) + 1/((a - b)^2*Sqrt[a + b*Cot[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$

$$= \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)}{2(a-b)}$$

$$= \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, c\right.}{2(a-b)^2}$$

$$= \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a\right.}}{(a-b)^2 b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}$$

Mathematica [C] time = 0.10, size = 69, normalized size = 0.84

$$\frac{3b\left(a+b\cot^2(x)\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\cot^2(x)+a}{a-b}\right)+a(a-b)}{3b(a-b)^2\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] (a*(a - b) + 3*b*(a + b*Cot[x]^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Co
t[x]^2)/(a - b)])/(3*(a - b)^2*b*(a + b*Cot[x]^2)^(3/2))
```

fricas [B] time = 0.68, size = 698, normalized size = 8.51

$$\left[\frac{3\left(a^2b+2\,ab^2+b^3+\left(a^2b-2\,ab^2+b^3\right)\cos\left(2\,x\right)^2-2\left(a^2b-b^3\right)\cos\left(2\,x\right)\right)\sqrt{a-b}\,\log\left(\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2\,x)-a-b}{\cos(2\,x)-1}}\right.}{6\left(a^5b-a^4b^2-2\,a^3b^3+2\,a^2b^4+ab^5-b^6+\left(a^5b-5\,a^4b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*b + 2*a*b^2 + b^3 + (a^2*b - 2*a*b^2 + b^3)*cos(2*x)^2 - 2*(a^
2*b - b^3)*cos(2*x))*sqrt(a - b)*log(sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a
 - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) + 2*(a^3 + a^2
*b + a*b^2 - 3*b^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cos(2*x)^2 - 2*(a^3 +
a^2*b - 2*a*b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))
/(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6 + (a^5*b - 5*a^4*b^
2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*cos(2*x)^2 - 2*(a^5*b - 3*a^4*
b^2 + 2*a^3*b^3 + 2*a^2*b^4 - 3*a*b^5 + b^6)*cos(2*x)), -1/3*(3*(a^2*b + 2*
a*b^2 + b^3 + (a^2*b - 2*a*b^2 + b^3)*cos(2*x)^2 - 2*(a^2*b - b^3)*cos(2*x)
)*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*
x) - 1))/(a - b)) - (a^3 + a^2*b + a*b^2 - 3*b^3 + (a^3 + a^2*b - 5*a*b^2 +
 3*b^3)*cos(2*x)^2 - 2*(a^3 + a^2*b - 2*a*b^2)*cos(2*x))*sqrt(((a - b)*cos(
2*x) - a - b)/(cos(2*x) - 1)))/(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a
*b^5 - b^6 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
cos(2*x)^2 - 2*(a^5*b - 3*a^4*b^2 + 2*a^3*b^3 + 2*a^2*b^4 - 3*a*b^5 + b^6)*
cos(2*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [A] time = 0.17, size = 88, normalized size = 1.07

$$\frac{1}{3b\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}+\frac{1}{(a-b)^2\sqrt{a+b\left(\cot^2(x)\right)}}+\frac{1}{3\,(a-b)\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*cot(x)^2)^(5/2),x)
```

```
[Out] 1/3/b/(a+b*cot(x)^2)^(3/2)+1/(a-b)^2/(a+b*cot(x)^2)^(1/2)+1/3/(a-b)/(a+b*co
t(x)^2)^(3/2)+1/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/
2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 4.24, size = 88, normalized size = 1.07

$$\frac{\frac{a}{3\,(a-b)}+\frac{b\,\left(b\cot(x)^2+a\right)}{(a-b)^2}}{b\,\left(b\cot(x)^2+a\right)^{3/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}\,\left(2\,a^2-4\,a\,b+2\,b^2\right)}{2\,(a-b)^{5/2}}\right)}{(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] (a/(3*(a - b)) + (b*(a + b*cot(x)^2))/(a - b)^2)/(b*(a + b*cot(x)^2)^(3/2))
 - atanh(((a + b*cot(x)^2)^(1/2)*(2*a^2 - 4*a*b + 2*b^2))/(2*(a - b)^(5/2))
)/(a - b)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] Integral(cot(x)**3/(a + b*cot(x)**2)**(5/2), x)
```

3.55 $\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(5/2)-1/3*cot(x)/(a-b
)/(a+b*cot(x)^2)^(3/2)-1/3*(2*a+b)*cot(x)/a/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 471, 527, 12, 377, 203}

$$-\frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(5/2) - Cot[x]/(3
*(a - b)*(a + b*Cot[x]^2)^(3/2)) - ((2*a + b)*Cot[x])/(3*a*(a - b)^2*Sqrt[a
 + b*Cot[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1-2x^2}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)}{3(a-b)}$$

$$= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{3a}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx\right.}{3a(a-b)^2}$$

$$= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx\right.}{(a-b)^2}$$

$$= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x,\right.}{(a-b)^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}} - \frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}}$$

Mathematica [C] time = 6.51, size = 200, normalized size = 2.13

$$\frac{\tan(x)\left(-\frac{35a\sin^2(x)\left(5a+2b\cot^2(x)\right)\left(a\csc^2(x)\left((a-4b)\cot^2(x)-3a\right)\sqrt{\frac{(a-b)\sin^2(x)\cos^2(x)\left(a+b\cot^2(x)\right)}{a^2}}+3\left(a+b\cot^2(x)\right)^2\sin^{-1}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\right)}{\sqrt{\frac{(a-b)\sin^2(x)\cos^2(x)\left(a+b\cot^2(x)\right)}{a^2}}} - \right.}{315a^3(a-b)^2\left(a+b\cot^2(x)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ((-12*(a - b)^3*Cos[x]^4*Cot[x]^2*(a + b*Cot[x]^2)*Hypergeometric2F1[2, 2,
9/2, ((a - b)*Cos[x]^2)/a] - (35*a*(5*a + 2*b*Cot[x]^2)*Sin[x]^2*(3*ArcSin[
```

```
Sqrt[((a - b)*Cos[x]^2)/a]]*(a + b*Cot[x]^2)^2 + a*(-3*a + (a - 4*b)*Cot[x]
^2)*Csc[x]^2*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]))/Sqrt[
((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2])*Tan[x])/(315*a^3*(a - b)
^2*(a + b*Cot[x]^2)^(3/2))
```

fricas [B] time = 1.35, size = 720, normalized size = 7.66

$$\left[-\frac{3\left(a^{3}+2\,a^{2}b+ab^{2}+\left(a^{3}-2\,a^{2}b+ab^{2}\right)\cos\left(2\,x\right)^{2}-2\left(a^{3}-ab^{2}\right)\cos\left(2\,x\right)\right)\sqrt{-a+b}\,\log\left(-2\left(a^{2}-2\,ab+b^{2}\right)\mathrm{co}\right.}{12\left(a^{6}-a^{5}b-2\,a^{4}b^{2}+2\,a^{3}b^{3}+a^{2}b^{4}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3 + 2*a^2*b + a*b^2 + (a^3 - 2*a^2*b + a*b^2)*cos(2*x)^2 - 2*(
a^3 - a*b^2)*cos(2*x))*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 +
 2*((a - b)*cos(2*x) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos
(2*x) - 1))*sin(2*x) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*x)) + 4*(3*a^3 - a
^2*b - a*b^2 - b^3 - (3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(2*x))*sqrt(((a - b
)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^6 - a^5*b - 2*a^4*b^2 + 2*
a^3*b^3 + a^2*b^4 - a*b^5 + (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^
2*b^4 - a*b^5)*cos(2*x)^2 - 2*(a^6 - 3*a^5*b + 2*a^4*b^2 + 2*a^3*b^3 - 3*a^
2*b^4 + a*b^5)*cos(2*x)), 1/6*(3*(a^3 + 2*a^2*b + a*b^2 + (a^3 - 2*a^2*b +
a*b^2)*cos(2*x)^2 - 2*(a^3 - a*b^2)*cos(2*x))*sqrt(a - b)*arctan(-sqrt(a -
b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*
x) - b)) - 2*(3*a^3 - a^2*b - a*b^2 - b^3 - (3*a^3 - 5*a^2*b + a*b^2 + b^3)
*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^6 -
 a^5*b - 2*a^4*b^2 + 2*a^3*b^3 + a^2*b^4 - a*b^5 + (a^6 - 5*a^5*b + 10*a^4*
b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cos(2*x)^2 - 2*(a^6 - 3*a^5*b + 2*a^4
*b^2 + 2*a^3*b^3 - 3*a^2*b^4 + a*b^5)*cos(2*x))]
```

giac [B] time = 2.40, size = 1025, normalized size = 10.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*a + b)*sgn(tan(1/2*x))/(a^3*sqrt(b) - 2*a^2*b^(3/2) + a*b^(5/2)) + 1
/3*((((2*a^10*b*sgn(tan(1/2*x)) - 15*a^9*b^2*sgn(tan(1/2*x)) + 48*a^8*b^3*s
gn(tan(1/2*x)) - 84*a^7*b^4*sgn(tan(1/2*x)) + 84*a^6*b^5*sgn(tan(1/2*x)) -
42*a^5*b^6*sgn(tan(1/2*x)) + 12*a^3*b^8*sgn(tan(1/2*x)) - 6*a^2*b^9*sgn(tan
(1/2*x)) + a*b^10*sgn(tan(1/2*x)))*tan(1/2*x)^2/(a^12 - 10*a^11*b + 45*a^10
*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7
+ 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10) + 3*(4*a^11*sgn(tan(1/2*x)) - 34*a^10
*b*sgn(tan(1/2*x)) + 127*a^9*b^2*sgn(tan(1/2*x)) - 272*a^8*b^3*sgn(tan(1/2*
x)) + 364*a^7*b^4*sgn(tan(1/2*x)) - 308*a^6*b^5*sgn(tan(1/2*x)) + 154*a^5*b
^6*sgn(tan(1/2*x)) - 32*a^4*b^7*sgn(tan(1/2*x)) - 8*a^3*b^8*sgn(tan(1/2*x))
 + 6*a^2*b^9*sgn(tan(1/2*x)) - a*b^10*sgn(tan(1/2*x)))/(a^12 - 10*a^11*b +
45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 120*a
^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10))*tan(1/2*x)^2 - 3*(4*a^11*sgn(
tan(1/2*x)) - 34*a^10*b*sgn(tan(1/2*x)) + 127*a^9*b^2*sgn(tan(1/2*x)) - 272
*a^8*b^3*sgn(tan(1/2*x)) + 364*a^7*b^4*sgn(tan(1/2*x)) - 308*a^6*b^5*sgn(ta
n(1/2*x)) + 154*a^5*b^6*sgn(tan(1/2*x)) - 32*a^4*b^7*sgn(tan(1/2*x)) - 8*a^
3*b^8*sgn(tan(1/2*x)) + 6*a^2*b^9*sgn(tan(1/2*x)) - a*b^10*sgn(tan(1/2*x)))
/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5
```

```
+ 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10))*tan(1/2*
x)^2 - (2*a^10*b*sgn(tan(1/2*x)) - 15*a^9*b^2*sgn(tan(1/2*x)) + 48*a^8*b^3*
sgn(tan(1/2*x)) - 84*a^7*b^4*sgn(tan(1/2*x)) + 84*a^6*b^5*sgn(tan(1/2*x)) -
 42*a^5*b^6*sgn(tan(1/2*x)) + 12*a^3*b^8*sgn(tan(1/2*x)) - 6*a^2*b^9*sgn(ta
n(1/2*x)) + a*b^10*sgn(tan(1/2*x)))/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a
^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8
 - 10*a^3*b^9 + a^2*b^10))/(b*tan(1/2*x)^4 + 4*a*tan(1/2*x)^2 - 2*b*tan(1/2
*x)^2 + b)^(3/2) - 2*arctan(-1/2*(sqrt(b)*tan(1/2*x)^2 - sqrt(b*tan(1/2*x)^
4 + 4*a*tan(1/2*x)^2 - 2*b*tan(1/2*x)^2 + b) + sqrt(b))/sqrt(a - b))/((a^2*
sgn(tan(1/2*x)) - 2*a*b*sgn(tan(1/2*x)) + b^2*sgn(tan(1/2*x)))*sqrt(a - b))
```

maple [B] time = 0.20, size = 166, normalized size = 1.77

$$-\frac{\cot(x)}{3a\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}-\frac{2\cot(x)}{3a^2\sqrt{a+b\left(\cot^2(x)\right)}}-\frac{b\cot(x)}{(a-b)^2\,a\sqrt{a+b\left(\cot^2(x)\right)}}-\frac{b\cot(x)}{3\,(a-b)\,a\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}-\frac{}{3\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+b*cot(x)^2)^(5/2),x)
```

```
[Out] -1/3*cot(x)/a/(a+b*cot(x)^2)^(3/2)-2/3/a^2*cot(x)/(a+b*cot(x)^2)^(1/2)-b/(a
-b)^2*cot(x)/a/(a+b*cot(x)^2)^(1/2)-1/3*b/(a-b)*cot(x)/a/(a+b*cot(x)^2)^(3/
2)-2/3*b/(a-b)/a^2*cot(x)/(a+b*cot(x)^2)^(1/2)+1/(a-b)^3*(b^4*(a-b))^(1/2)/
b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)^2}{\left(b\cot(x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] int(cot(x)^2/(a + b*cot(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] Integral(cot(x)**2/(a + b*cot(x)**2)**(5/2), x)
```

3.56 $\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)-1/3/(a-b)/(a+b*cot(x)
^2)^(3/2)-1/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 51, 63, 208}

$$-\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(5/2) - 1/(3*(a - b)*(a +
 b*Cot[x]^2)^(3/2)) - 1/((a - b)^2*Sqrt[a + b*Cot[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$

$$= -\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)}{2(a-b)}$$

$$= -\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \text{c}\ldots\right.}{2(a-b)^2}$$

$$= -\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a\ldots}\right.}{(a-b)^2 b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} - \frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}$$

Mathematica [C] time = 0.04, size = 47, normalized size = 0.60

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\cot^2(x)+a}{a-b}\right)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Cot[x]^2)/(a - b)]/((a - b)*(a
 + b*Cot[x]^2)^(3/2))
```

fricas [B] time = 0.83, size = 627, normalized size = 8.04

$$\left[\frac{3\left(\left(a^2-2ab+b^2\right)\cos(2x)^2+a^2+2ab+b^2-2\left(a^2-b^2\right)\cos(2x)\right)\sqrt{a-b}\,\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,(\ldots\right.}{6\left(a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+\left(a^5-5a^4b+10a^3b^2-\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^2 - 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)
*cos(2*x))*sqrt(a - b)*log(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(co
s(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) - 4*(2*(a^2 - 2*a*b + b
^2)*cos(2*x)^2 + 2*a^2 - a*b - b^2 - (4*a^2 - 5*a*b + b^2)*cos(2*x))*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b
^5)*cos(2*x)^2 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*
cos(2*x)), 1/3*(3*((a^2 - 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(
a^2 - b^2)*cos(2*x))*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*
x) - a - b)/(cos(2*x) - 1))/(a - b)) - 2*(2*(a^2 - 2*a*b + b^2)*cos(2*x)^2
+ 2*a^2 - a*b - b^2 - (4*a^2 - 5*a*b + b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1)))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*cos(2*x)^2
- 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cos(2*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [A] time = 0.14, size = 75, normalized size = 0.96

$$-\frac{1}{\left(a-b\right)^{2} \sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}-\frac{1}{3\left(a-b\right)\left(a+b\left(\cot^{2}\left(x\right)\right)\right)^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\left(a-b\right)^{2} \sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^2)^(5/2),x)
```

```
[Out] -1/(a-b)^2/(a+b*cot(x)^2)^(1/2)-1/3/(a-b)/(a+b*cot(x)^2)^(3/2)-1/(a-b)^2/(-
a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 4.46, size = 82, normalized size = 1.05

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,{\mathrm{cot}\left(x\right)}^2+a}\,\left(2\,a^2-4\,a\,b+2\,b^2\right)}{2\,{\left(a-b\right)}^{5/2}}\right)}{{\left(a-b\right)}^{5/2}}-\frac{\frac{1}{3\,\left(a-b\right)}+\frac{b\,{\mathrm{cot}\left(x\right)}^2+a}{{\left(a-b\right)}^2}}{{\left(b\,{\mathrm{cot}\left(x\right)}^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*cot(x)^2)^(1/2)*(2*a^2 - 4*a*b + 2*b^2))/(2*(a - b)^(5/2)))/(
a - b)^(5/2) - (1/(3*(a - b)) + (a + b*cot(x)^2)/(a - b)^2)/(a + b*cot(x)^2
)^(3/2)
```

sympy [**A**]   time = 16.77, size = 70, normalized size = 0.90

$$-\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{\frac{3}{2}}}-\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] -1/(3*(a - b)*(a + b*cot(x)**2)**(3/2)) - 1/((a - b)**2*sqrt(a + b*cot(x)**
2)) - atan(sqrt(a + b*cot(x)**2)/sqrt(-a + b))/(sqrt(-a + b)*(a - b)**2)
```

3.57 $\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}} dx$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b(2a-b)}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(5/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))/(a-b)^(5/2)+1/3*b/a/(a-b)/(a+b*cot(x)^2)^(3/2)+(2*a-b)*b/a^2/(
a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3670, 446, 85, 152, 156, 63, 208}

$$\frac{b(2a-b)}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/a^(5/2) - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/(a - b)^(5/2) + b/(3*a*(a - b)*(a + b*Cot[x]^2)^(3/2)) + ((2
*a - b)*b)/(a^2*(a - b)^2*Sqrt[a + b*Cot[x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$

$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)}{2a(a-b)}$$

$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a-b)^2+\frac{1}{2}(2a-b)b}{x(1+x)\sqrt{a+bx}}\right.}{a^2(a-b)^2}$$

$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{co}\right.}{2a^2}$$

$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{a^2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{}{a^2(a-}$$

Mathematica [C] time = 0.05, size = 78, normalized size = 0.66

$$\frac{a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\cot^2(x)+a}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\cot^2(x)}{a}+1\right)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] (a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Cot[x]^2)/(a - b)] + (-a + b)*Hy
pergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cot[x]^2)/a])/(3*a*(a - b)*(a + b*Cot
[x]^2)^(3/2))
```

fricas [B] time = 0.82, size = 1531, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*tan(x)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sq
rt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2
 + b) + 3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*log(((2*a
 - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b
)/(tan(x)^2 + 1)) + 2*((7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4 + 3*(2*a
^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^6
*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b
^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2), -1/6*
(6*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*arctan(-sqrt(-a
 + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) - 3*(a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^4 + 2*(a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(
a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b) - 2*((7*a^4*b - 11*a^3*b^2
 + 4*a^2*b^3)*tan(x)^4 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((
a*tan(x)^2 + b)/tan(x)^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^
8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*
b^3 - a^4*b^4)*tan(x)^2), -1/6*(6*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(
x)^2)/a) - 3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*log(((
2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2
+ b)/(tan(x)^2 + 1)) - 2*((7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4 + 3*(
2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(
a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^
5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2), -1
/3*(3*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a
^2*b^3)*tan(x)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) + 3*(a^5*tan(x)^4 +
 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(
x)^2 + b)/tan(x)^2)/(a - b)) - ((7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4
 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)
^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^
2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^
2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, integration of abs or sign assumes constant sign b
y intervals (correct if the argument is real):Check [abs(t_nostep)]Evaluati
on time: 0.79Error: Bad Argument Type
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(a+b\left(\cot^2(x)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cot(x)^2)^(5/2),x)
```

```
[Out] int(tan(x)/(a+b*cot(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(b\cot(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*cot(x)^2 + a)^(5/2), x)
```

mupad [B] time = 1.05, size = 2817, normalized size = 23.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] atanh((2*a^5*b^13*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4
*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b
^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (22*a^6*b^
12*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5
*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b
^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (110*a^7*b^11*(a + b/tan(
```

x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (330*a^8*b^10*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (660*a^9*b^9*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (922*a^10*b^8*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (912*a^11*b^7*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (630*a^12*b^6*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (290*a^13*b^5*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (80*a^14*b^4*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (10*a^15*b^3*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)))/(a^5)^(1/2) - (b/(3*(a*b - a^2)) - (b*(a + b/tan(x)^2)*(2*a - b))/(a*b - a^2)^2)/(a + b/tan(x)^2)^(3/2) + (atan((((((a + b/tan(x)^2)^(1/2)*(2*a^6*b^12 - 20*a^7*b^11 + 90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b^7 + 450*a^12*b^6 - 280*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2))/2 - (((a - b)^5)^(1/2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^11*b^10 + 644*a^12*b^9 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a^16*b^5 - 54*a^17*b^4 + 6*a^18*b^3 - ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1/2)*(8*a^10*b^13 - 96*a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^10 + 3600*a^14*b^9 - 5376*a^15*b^8 + 5712*a^16*b^7 - 4320*a^17*b^6 + 2280*a^18*b^5 - 800*a^19*b^4 + 168*a^20*b^3 - 16*a^21*b^2))/(4*(a - b)^5)))/(2*(a - b)^5))*((a - b)^5)^(1/2)*1i)/(a - b)^5 + ((((a + b/tan(x)^2)^(1/2)*(2*a^6*b^12 - 20*a^7*b^11 + 90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b^7 + 450*a^12*b^6 - 280*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2))/2 + (((a - b)^5)^(1/2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^11*b^10 + 644*a^12*b^9 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a^16*b^5 - 54*a^17*b^4 + 6*a^18*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1/2)*(8*a^10*b^13 - 96*a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^10 + 3600*a^14*b^9 - 5376*a^15*b^8 + 5712*a^16*b^7 - 4320*a^17*b^6 + 2280*a^18*b^5 - 800*a^19*b^4 + 168*a^20*b^3 - 16*a^21*b^2))/(4*(a - b)^5)))/(2*(a - b)^5))*((a - b)^5)^(1/2)*1i)/(a - b)^5)/(2*a^6*b^10 - 16*a^7*b^9 + 54*a^8*b^8 - 100*a^9*b^7 + 110*a^10*b^6 - 72*a^11*b^5 + 26*a^12*b^4 - 4*a^13*b^3 + ((((a + b/tan(x)^2)^(1/2)*(2*a^6*b^12 - 20*a^7*b^11 + 90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b^7 + 450*a^12*b^6 - 280*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2))/2 - (((a - b)^5)^(1/2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^11*b^10 + 644*a^12*b^9 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a^16*b^5 - 54*a^17*b^4 + 6*a^18*b^3 - ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1/2)*(8*a^10*b^13 - 96*a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^10 + 3600*a^14*b^9 - 5376*a^15*b^8 + 5712*a^16*b^7 - 4320*a^17*b^6 + 2280*a^18*b^5 - 800*a^19*b^4 + 168*a^20*b^3 - 16*a^21*b^2))/(4*(a - b)^5)))/(2*(a - b)^5))*((a - b)^5)^(1/2))/(a - b)^5 - ((((a + b/tan(x)^2)^(1/2)*(2*a^6*b^12 - 20*a^7*b^11 + 90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b^7 + 450*a^12*b^6 - 280*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2))/2 + (((a - b)^5)^(1/2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^11*b^10 + 644*a^12*b^9 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a^16*b^5 - 54*a^

```
17*b^4 + 6*a^18*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1/2)*(8*a^10*b^1
3 - 96*a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^10 + 3600*a^14*b^9 - 5376*a^
15*b^8 + 5712*a^16*b^7 - 4320*a^17*b^6 + 2280*a^18*b^5 - 800*a^19*b^4 + 168
*a^20*b^3 - 16*a^21*b^2))/(4*(a - b)^5)))/(2*(a - b)^5))*((a - b)^5)^(1/2))
/(a - b)^5))*((a - b)^5)^(1/2)*1i)/(a - b)^5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan (x)}{\left(a + b\cot^2 (x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] Integral(tan(x)/(a + b*cot(x)**2)**(5/2), x)
```

3.58 $\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=141

$$\frac{(a-4b)(3a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{3a^3(a-b)^2}+\frac{b(7a-4b)\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}+\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x\right.}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(5/2)+1/3*b*tan(x)/a/
(a-b)/(a+b*cot(x)^2)^(3/2)+1/3*(7*a-4*b)*b*tan(x)/a^2/(a-b)^2/(a+b*cot(x)^2
)^(1/2)+1/3*(a-4*b)*(3*a-2*b)*(a+b*cot(x)^2)^(1/2)*tan(x)/a^3/(a-b)^2
```

Rubi [A] time = 0.24, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 472, 579, 583, 12, 377, 203}

$$\frac{(a-4b)(3a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{3a^3(a-b)^2}+\frac{b(7a-4b)\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}+\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(5/2) + (b*Tan[x]
)/(3*a*(a - b)*(a + b*Cot[x]^2)^(3/2)) + ((7*a - 4*b)*b*Tan[x])/(3*a^2*(a -
 b)^2*Sqrt[a + b*Cot[x]^2]) + ((a - 4*b)*(3*a - 2*b)*Sqrt[a + b*Cot[x]^2]*T
an[x])/(3*a^3*(a - b)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{3a-4b-4bx^2}{x^2\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)}{3a(a-b)}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{(a-4b)(3a-2b)-2(7a-4}{x^2\left(1+x^2\right)\sqrt{a+bx^2}}\right.}{3a^2(a-b)}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-4b)(3a-2b)\sqrt{a+b\,\text{co}}}{3a^3(a-b)^2}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-4b)(3a-2b)\sqrt{a+b\,\text{co}}}{3a^3(a-b)^2}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-4b)(3a-2b)\sqrt{a+b\,\text{co}}}{3a^3(a-b)^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}} + \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-}{}$$

Mathematica [C] time = 8.02, size = 1450, normalized size = 10.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] (Sin[x]^2*((-16*b^3*(Cot[x] + Cot[x]^3)^2)/(a*(a - b)^2) + (40*b*Csc[x]^2)/
(a - b) + (160*b^2*Cot[x]^2*Csc[x]^2)/(3*a*(a - b)) + (64*b^3*Cot[x]^4*Csc[
x]^2)/(3*a^2*(a - b)) - (40*b^2*Csc[x]^4)/(a - b)^2 + (92*(a - b)*Cos[x]^2*
Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(105*a) + (124*(a - b)*
b*Cos[x]^2*Cot[x]^2*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(35
*a^2) + (152*(a - b)*b^2*Cos[x]^2*Cot[x]^4*Hypergeometric2F1[2, 2, 9/2, ((a
 - b)*Cos[x]^2)/a])/(35*a^3) + (176*(a - b)*b^3*Cos[x]^2*Cot[x]^6*Hypergeom
etric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(105*a^4) + (24*(a - b)*Cos[x]^2
*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a) + (16
*(a - b)*b*Cos[x]^2*Cot[x]^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b
)*Cos[x]^2)/a])/(7*a^2) + (88*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricP
FQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a^3) + (32*(a - b)*b^3*C
os[x]^2*Cot[x]^6*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/
a])/(35*a^4) + (16*(a - b)*Cos[x]^2*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1,
9/2}, ((a - b)*Cos[x]^2)/a])/(105*a) + (16*(a - b)*b*Cos[x]^2*Cot[x]^2*Hype
rgeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a^2) +
(16*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/
2}, ((a - b)*Cos[x]^2)/a])/(35*a^3) + (16*(a - b)*b^3*Cos[x]^2*Cot[x]^6*Hyp
ergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x]^2)/a])/(105*a^4)
```

```
+ (20*a*Sec[x]^2)/(3*(a - b)) - (30*a*b*Csc[x]^2*Sec[x]^2)/(a - b)^2 - (5*a
^2*Sec[x]^4)/(a - b)^2 + (5*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]])/((((a - b)*
Cos[x]^2)/a)^(5/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) + (30*b*ArcSin[Sqrt
[((a - b)*Cos[x]^2)/a]]*Cot[x]^2)/(a*(((a - b)*Cos[x]^2)/a)^(5/2)*Sqrt[((a
+ b*Cot[x]^2)*Sin[x]^2)/a]) + (40*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Co
t[x]^4)/(a^2*(((a - b)*Cos[x]^2)/a)^(5/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/
a]) + (16*b^3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^6)/(a^3*(((a - b)*C
os[x]^2)/a)^(5/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) + (5*ArcSin[Sqrt[((a
 - b)*Cos[x]^2)/a]])/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]
 + (30*b*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^2)/(a*Sqrt[((a - b)*Cos[
x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) + (40*b^2*ArcSin[Sqrt[((a - b)*Cos[x]
^2)/a]]*Cot[x]^4)/(a^2*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^
2]) + (16*b^3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^6)/(a^3*Sqrt[((a -
b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) - (60*b*ArcSin[Sqrt[((a - b)*C
os[x]^2)/a]]*Csc[x]^2)/((a - b)*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin
[x]^2)/a^2]) - (80*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^2*Csc[x]^2
)/(a*(a - b)*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) - (32*
b^3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^4*Csc[x]^2)/(a^2*(a - b)*Sqrt
[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) - (10*a*ArcSin[Sqrt[((a
 - b)*Cos[x]^2)/a]]*Sec[x]^2)/((a - b)*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]
^2)*Sin[x]^2)/a^2]))*Tan[x])/(a^2*Sqrt[a + b*Cot[x]^2]*(1 + (b*Cot[x]^2)/a)
)
```

fricas [B] time = 0.70, size = 647, normalized size = 4.59

$$\Bigg[-\frac{3\,(a^5\tan(x)^4+2\,a^4b\tan(x)^2+a^3b^2)\sqrt{-a+b}\,\log\Big(-\frac{a^2\tan(x)^4-2\,(3\,a^2-4\,ab)\tan(x)^2+a^2-8\,ab+8\,b^2+4\,(a\tan(x)^3-(a-2\,b)\tan(}{\tan(x)^4+2\,\tan(x)^2+1}}{12\,(a^6b^2-3\,a^5b^3+3\,a^4b^4-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*log(-(a^
2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)
^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x
)^4 + 2*tan(x)^2 + 1)) - 4*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^
5 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(x)^3 + (3*a^3*b^2 -
17*a^2*b^3 + 22*a*b^4 - 8*b^5)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^
6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*
b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2), 1/6*
(3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*arctan(2*sqrt(a
- b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + 2*(3*
(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^5 + 3*(2*a^4*b - 9*a^3*b^2 + 1
1*a^2*b^3 - 4*a*b^4)*tan(x)^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)
*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4
- a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a
^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2)]
```

giac [B] time = 4.94, size = 1242, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(8*a*b^2 - 5*b^3)*sgn(tan(1/2*x))/(a^5*sqrt(b) - 2*a^4*b^(3/2) + a^3*b^
(5/2)) + 1/3*((((8*a^16*b^3*sgn(tan(1/2*x)) - 69*a^15*b^4*sgn(tan(1/2*x)) +
```

```
 264*a^14*b^5*sgn(tan(1/2*x)) - 588*a^13*b^6*sgn(tan(1/2*x)) + 840*a^12*b^7
*sgn(tan(1/2*x)) - 798*a^11*b^8*sgn(tan(1/2*x)) + 504*a^10*b^9*sgn(tan(1/2*
x)) - 204*a^9*b^10*sgn(tan(1/2*x)) + 48*a^8*b^11*sgn(tan(1/2*x)) - 5*a^7*b^
12*sgn(tan(1/2*x)))*tan(1/2*x)^2/(a^20 - 10*a^19*b + 45*a^18*b^2 - 120*a^17
*b^3 + 210*a^16*b^4 - 252*a^15*b^5 + 210*a^14*b^6 - 120*a^13*b^7 + 45*a^12*
b^8 - 10*a^11*b^9 + a^10*b^10) + 3*(12*a^17*b^2*sgn(tan(1/2*x)) - 112*a^16*
b^3*sgn(tan(1/2*x)) + 469*a^15*b^4*sgn(tan(1/2*x)) - 1160*a^14*b^5*sgn(tan(
1/2*x)) + 1876*a^13*b^6*sgn(tan(1/2*x)) - 2072*a^12*b^7*sgn(tan(1/2*x)) + 1
582*a^11*b^8*sgn(tan(1/2*x)) - 824*a^10*b^9*sgn(tan(1/2*x)) + 280*a^9*b^10*
sgn(tan(1/2*x)) - 56*a^8*b^11*sgn(tan(1/2*x)) + 5*a^7*b^12*sgn(tan(1/2*x)))
/(a^20 - 10*a^19*b + 45*a^18*b^2 - 120*a^17*b^3 + 210*a^16*b^4 - 252*a^15*b
^5 + 210*a^14*b^6 - 120*a^13*b^7 + 45*a^12*b^8 - 10*a^11*b^9 + a^10*b^10))*
tan(1/2*x)^2 - 3*(12*a^17*b^2*sgn(tan(1/2*x)) - 112*a^16*b^3*sgn(tan(1/2*x)
) + 469*a^15*b^4*sgn(tan(1/2*x)) - 1160*a^14*b^5*sgn(tan(1/2*x)) + 1876*a^1
3*b^6*sgn(tan(1/2*x)) - 2072*a^12*b^7*sgn(tan(1/2*x)) + 1582*a^11*b^8*sgn(t
an(1/2*x)) - 824*a^10*b^9*sgn(tan(1/2*x)) + 280*a^9*b^10*sgn(tan(1/2*x)) -
56*a^8*b^11*sgn(tan(1/2*x)) + 5*a^7*b^12*sgn(tan(1/2*x)))/(a^20 - 10*a^19*b
 + 45*a^18*b^2 - 120*a^17*b^3 + 210*a^16*b^4 - 252*a^15*b^5 + 210*a^14*b^6
- 120*a^13*b^7 + 45*a^12*b^8 - 10*a^11*b^9 + a^10*b^10))*tan(1/2*x)^2 - (8*
a^16*b^3*sgn(tan(1/2*x)) - 69*a^15*b^4*sgn(tan(1/2*x)) + 264*a^14*b^5*sgn(t
an(1/2*x)) - 588*a^13*b^6*sgn(tan(1/2*x)) + 840*a^12*b^7*sgn(tan(1/2*x)) -
798*a^11*b^8*sgn(tan(1/2*x)) + 504*a^10*b^9*sgn(tan(1/2*x)) - 204*a^9*b^10*
sgn(tan(1/2*x)) + 48*a^8*b^11*sgn(tan(1/2*x)) - 5*a^7*b^12*sgn(tan(1/2*x)))
/(a^20 - 10*a^19*b + 45*a^18*b^2 - 120*a^17*b^3 + 210*a^16*b^4 - 252*a^15*b
^5 + 210*a^14*b^6 - 120*a^13*b^7 + 45*a^12*b^8 - 10*a^11*b^9 + a^10*b^10))/
(b*tan(1/2*x)^4 + 4*a*tan(1/2*x)^2 - 2*b*tan(1/2*x)^2 + b)^(3/2) - 2*arctan
(-1/2*(sqrt(b)*tan(1/2*x)^2 - sqrt(b*tan(1/2*x)^4 + 4*a*tan(1/2*x)^2 - 2*b*
tan(1/2*x)^2 + b) + sqrt(b))/sqrt(a - b))/((a^2*sgn(tan(1/2*x)) - 2*a*b*sgn
(tan(1/2*x)) + b^2*sgn(tan(1/2*x)))*sqrt(a - b)) + 4*(sqrt(b)*tan(1/2*x)^2
- sqrt(b*tan(1/2*x)^4 + 4*a*tan(1/2*x)^2 - 2*b*tan(1/2*x)^2 + b) + sqrt(b))
/(((sqrt(b)*tan(1/2*x)^2 - sqrt(b*tan(1/2*x)^4 + 4*a*tan(1/2*x)^2 - 2*b*tan
(1/2*x)^2 + b))^2 - 2*(sqrt(b)*tan(1/2*x)^2 - sqrt(b*tan(1/2*x)^4 + 4*a*tan
(1/2*x)^2 - 2*b*tan(1/2*x)^2 + b))*sqrt(b) - 4*a + b)*a^2*sgn(tan(1/2*x)))
```

maple [B] time = 1.30, size = 1040, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*cot(x)^2)^(5/2),x)
```

```
[Out] 1/3*(-1+cos(x))^2*(cos(x)+1)^2*(a*cos(x)^2-b*cos(x)^2-a)*(3*cos(x)^4*(-(a*c
os(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos
(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2
)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^4-3*cos(x)^4*(-(a*cos(
x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)
^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(
-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^3*b-3*cos(x)^4*(-a+b)^(1/
2)*a^4+12*cos(x)^4*(-a+b)^(1/2)*a^3*b-27*cos(x)^4*(-a+b)^(1/2)*a^2*b^2+26*c
os(x)^4*(-a+b)^(1/2)*a*b^3-8*cos(x)^4*(-a+b)^(1/2)*b^4+3*cos(x)^3*(-(a*cos(
x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)
^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(
-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^4-3*cos(x)^3*(-(a*cos(x)^
2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-
b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a
*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^3*b-3*cos(x)^2*(-(a*cos(x)^2
-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b
*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*
cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^4+6*cos(x)^2*(-a+b)^(1/2)*a^4
-18*cos(x)^2*(-a+b)^(1/2)*a^3*b+27*cos(x)^2*(-a+b)^(1/2)*a^2*b^2-12*cos(x)^
```

```
2*(-a+b)^(1/2)*a*b^3-3*cos(x)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/
2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)
-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(a*cos(x)^2-b*cos(x)^2-a)/(cos(x)+1
)^2)^(1/2))*a^4-3*(-a+b)^(1/2)*a^4+6*(-a+b)^(1/2)*a^3*b-3*(-a+b)^(1/2)*a^2*
b^2)*b^2/cos(x)/((a*cos(x)^2-b*cos(x)^2-a)/(-1+cos(x)^2))^(5/2)/sin(x)^9/((
a*(a-b))^(1/2)-a+b)^2/a^3/((a*(a-b))^(1/2)+a-b)^2/(-a+b)^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^2}{\left(b\cot(x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*cot(x)^2)^(5/2),x)
[Out] int(tan(x)^2/(a + b*cot(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*cot(x)**2)**(5/2),x)
[Out] Integral(tan(x)**2/(a + b*cot(x)**2)**(5/2), x)
```

3.59 $\int \frac{1}{1+\cot^3(x)} \, dx$

Optimal. Leaf size=37

$$\frac{x}{2}+\frac{1}{2}\log(\sin(x))+\frac{1}{3}\log\left(\cot^2(x)-\cot(x)+1\right)-\frac{1}{6}\log(\cot(x)+1)$$

```
[Out] 1/2*x-1/6*ln(1+cot(x))+1/3*ln(1-cot(x)+cot(x)^2)+1/2*ln(sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3661, 2074, 635, 203, 260, 628}

$$\frac{x}{2}+\frac{1}{2}\log(\sin(x))+\frac{1}{3}\log\left(\cot^2(x)-\cot(x)+1\right)-\frac{1}{6}\log(\cot(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[x]^3)^(-1),x]
```

```
[Out] x/2 - Log[1 + Cot[x]]/6 + Log[1 - Cot[x] + Cot[x]^2]/3 + Log[Sin[x]]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+\cot^3(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(1+x^3\right)}\,dx, x, \cot(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx, x, \cot(x)\right)\\
&= -\frac{1}{6}\log(1+\cot(x))-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-2x}{1-x+x^2}\,dx, x, \cot(x)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \cot\right.\\
&= -\frac{1}{6}\log(1+\cot(x))+\frac{1}{3}\log\left(1-\cot(x)+\cot^2(x)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(x)\right)-\frac{1}{2}\\
&= \frac{x}{2}-\frac{1}{6}\log(1+\cot(x))+\frac{1}{3}\log\left(1-\cot(x)+\cot^2(x)\right)+\frac{1}{2}\log(\sin(x))
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 57, normalized size = 1.54

$$\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\left(-\frac{1}{4}-\frac{i}{4}\right)\log(-\tan(x)+i)-\left(\frac{1}{4}-\frac{i}{4}\right)\log(\tan(x)+i)-\frac{1}{6}\log(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[x]^3)^(-1),x]
```

```
[Out] (-1/4 - I/4)*Log[I - Tan[x]] - (1/4 - I/4)*Log[I + Tan[x]] - Log[1 + Tan[x]
]/6 + Log[1 - Tan[x] + Tan[x]^2]/3
```

fricas [A] time = 0.46, size = 24, normalized size = 0.65

$$\frac{1}{2}x-\frac{1}{12}\log\left(\sin\left(2x\right)+1\right)+\frac{1}{3}\log\left(-\frac{1}{2}\sin\left(2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/12*log(sin(2*x) + 1) + 1/3*log(-1/2*sin(2*x) + 1)
```

giac [A] time = 0.42, size = 34, normalized size = 0.92

$$\frac{1}{2}x+\frac{1}{3}\log\left(\tan(x)^2-\tan(x)+1\right)-\frac{1}{4}\log\left(\tan(x)^2+1\right)-\frac{1}{6}\log\left(|\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(ab
s(tan(x) + 1))
```

maple [A] time = 0.15, size = 37, normalized size = 1.00

$$\frac{\ln\left(1-\cot(x)+\cot^2(x)\right)}{3}-\frac{\ln\left(1+\cot(x)\right)}{6}-\frac{\ln\left(1+\cot^2(x)\right)}{4}-\frac{\pi}{4}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cot(x)^3),x)
```

```
[Out] 1/3*ln(1-cot(x)+cot(x)^2)-1/6*ln(1+cot(x))-1/4*ln(1+cot(x)^2)-1/4*Pi+1/2*x
```

maxima [A] time = 0.42, size = 33, normalized size = 0.89

$$\frac{1}{2}\,x+\frac{1}{3}\,\log\left(\tan(x)^{2}-\tan(x)+1\right)-\frac{1}{4}\,\log\left(\tan(x)^{2}+1\right)-\frac{1}{6}\,\log\left(\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^3),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(ta
n(x) + 1)
```

mupad [B] time = 0.72, size = 37, normalized size = 1.00

$$x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)-\frac{\ln\left(12\,\mathrm{e}^{x\,2\mathrm{i}}+12\mathrm{i}\right)}{6}+\frac{\ln\left(\mathrm{e}^{x\,4\mathrm{i}}-1-\mathrm{e}^{x\,2\mathrm{i}}\,4\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^3 + 1),x)
```

```
[Out] x*(1/2 - 1i/2) - log(12*exp(x*2i) + 12i)/6 + log(exp(x*4i) - exp(x*2i)*4i -
 1)/3
```

sympy [A] time = 0.29, size = 34, normalized size = 0.92

$$\frac{x}{2}-\frac{\log\left(\tan\left(x\right)+1\right)}{6}-\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{4}+\frac{\log\left(\tan^{2}\left(x\right)-\tan\left(x\right)+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)**3),x)
```

```
[Out] x/2 - log(tan(x) + 1)/6 - log(tan(x)**2 + 1)/4 + log(tan(x)**2 - tan(x) + 1
)/3
```

3.60 $\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx$

Optimal. Leaf size=90

$$-\frac{1}{2}\sqrt{a+b\cot^4(x)}+\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)$$

```
[Out] 1/2*arctanh(cot(x)^2*b^(1/2)/(a+b*cot(x)^4)^(1/2))*b^(1/2)+1/2*arctanh((a-b
*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))*(a+b)^(1/2)-1/2*(a+b*cot(x)^4)
^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 735, 844, 217, 206, 725}

$$-\frac{1}{2}\sqrt{a+b\cot^4(x)}+\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Cot[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]])/2 + (Sqrt[a + b]
*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])])/2 - Sqrt[a +
 b*Cot[x]^4]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx &= -\text{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1+x^2}\,dx, x, \cot(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x}\,dx, x, \cot^2(x)\right)\right)\\ &= -\frac{1}{2}\sqrt{a+b\cot^4(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)\\ &= -\frac{1}{2}\sqrt{a+b\cot^4(x)} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right) - \frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{}{(}\right.\\ &= -\frac{1}{2}\sqrt{a+b\cot^4(x)} - \frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{2}b\,\text{S}\\ &= \frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right) - \frac{1}{2}\sqrt{}\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.96

$$\frac{1}{2}\left(-\sqrt{a+b\cot^4(x)}+\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Cot[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + Sqrt[a + b]*Arc
Tanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])] - Sqrt[a + b*Cot[
x]^4])/2
```

fricas [B] time = 0.83, size = 1063, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2
 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b
)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) -
```

```
(a^2 - b^2)*cos(2*x)) + 1/4*sqrt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2*
x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(c
os(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2
- 2*cos(2*x) + 1)) - 1/2*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a
+ b)/(cos(2*x)^2 - 2*cos(2*x) + 1)), 1/2*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a
+ b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)
)*(cos(2*x) - 1)/(b*cos(2*x) + b)) + 1/4*sqrt(a + b)*log(1/2*(a^2 + 2*a*b +
 b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*
x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a +
 b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)) - 1/2*sqrt(((a +
 b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))
, -1/2*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt
(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2
 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2
*(a^2 - b^2)*cos(2*x))) + 1/4*sqrt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2
*x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(
cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2
 - 2*cos(2*x) + 1)) - 1/2*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a
 + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)), -1/2*sqrt(-a - b)*arctan(((a + b)*cos
(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(
a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2
)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x))) + 1/2*sqrt(-b)*
arctan(sqrt(-b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos
(2*x)^2 - 2*cos(2*x) + 1))*(cos(2*x) - 1)/(b*cos(2*x) + b)) - 1/2*sqrt(((a
+ b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)
)]
```

giac [B] time = 0.45, size = 204, normalized size = 2.27

$$-\frac{b\arctan\left(-\frac{\sqrt{a+b}\,\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{1}{2}\,\sqrt{a+b}\,\log\left(\left|-\left(\sqrt{a+b}\,\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -b*arctan(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x
)^2 + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*sin(x)
^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))*(a + b) + sqrt(a + b
)*b)) - ((sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^
2 + b))*b - sqrt(a + b)*b)/((sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin
(x)^4 - 2*b*sin(x)^2 + b))^2 - b)
```

maple [A] time = 0.35, size = 139, normalized size = 1.54

$$-\frac{\sqrt{\left(1+\cot^2(x)\right)^2 b-2\left(1+\cot^2(x)\right)b+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{\left(1+\cot^2(x)\right)b-b}{\sqrt{b}}+\sqrt{\left(1+\cot^2(x)\right)^2 b-2\left(1+\cot^2(x)\right)b-\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^4)^(1/2),x)
```

```
[Out] -1/2*((1+cot(x)^2)^2*b-2*(1+cot(x)^2)*b+a+b)^(1/2)+1/2*b^(1/2)*ln(((1+cot(x
)^2)*b-b)/b^(1/2)+((1+cot(x)^2)^2*b-2*(1+cot(x)^2)*b+a+b)^(1/2))+1/2*(a+b)^
(1/2)*ln((2*a+2*b-2*(1+cot(x)^2)*b+2*(a+b)^(1/2)*((1+cot(x)^2)^2*b-2*(1+cot
(x)^2)*b+a+b)^(1/2))/(1+cot(x)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cot(x)^4 + a}\, \cot(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^4 + a)*cot(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x) \sqrt{b \cot(x)^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)*(a + b*cot(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cot^4(x)} \cot(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**4)*cot(x), x)
```

3.61 $\int \cot(x) \left(a + b \cot^4(x)\right)^{3/2} dx$

Optimal. Leaf size=126

$$-\frac{1}{6} \left(a + b \cot^4(x)\right)^{3/2} - \frac{1}{4} \left(2(a+b) - b\cot^2(x)\right) \sqrt{a + b\cot^4(x)} + \frac{1}{2}(a+b)^{3/2} \tanh^{-1}\left(\frac{a - b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{4}$$

```
[Out] 1/2*(a+b)^(3/2)*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))-1/
6*(a+b*cot(x)^4)^(3/2)+1/4*(3*a+2*b)*arctanh(cot(x)^2*b^(1/2)/(a+b*cot(x)^4
)^(1/2))*b^(1/2)-1/4*(2*a+2*b-b*cot(x)^2)*(a+b*cot(x)^4)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3670, 1248, 735, 815, 844, 217, 206, 725}

$$-\frac{1}{6} \left(a + b \cot^4(x)\right)^{3/2} - \frac{1}{4} \left(2(a+b) - b\cot^2(x)\right) \sqrt{a + b\cot^4(x)} + \frac{1}{2}(a+b)^{3/2} \tanh^{-1}\left(\frac{a - b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + b*Cot[x]^4)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]])/4 +
((a + b)^(3/2)*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]
)/2 - ((2*(a + b) - b*Cot[x]^2)*Sqrt[a + b*Cot[x]^4])/4 - (a + b*Cot[x]^4)^
(3/2)/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx = -\text{Subst}\left(\int \frac{x\left(a+bx^4\right)^{3/2}}{1+x^2}\,dx, x, \cot(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x}\,dx, x, \cot^2(x)\right)\right)$$

$$= -\frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}\,dx, x, \cot^2(x)\right)$$

$$= -\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)} - \frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} - \frac{\text{Subst}\left(\int \frac{ab(2a\cdots}{(1\cdots}\right.}{\cdots}$$

$$= -\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)} - \frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} - \frac{1}{2}(a+b)^2\,\text{Subs}\cdots$$

$$= -\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)} - \frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} + \frac{1}{2}(a+b)^2\,\text{Subs}\cdots$$

$$= \frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\cot^2(x\cdots}{\sqrt{a+b}\,\sqrt{a+b\,\text{co}\cdots}}\right.$$

Mathematica [A] time = 4.40, size = 167, normalized size = 1.33

$$\frac{1}{12}\left(-\sqrt{a+b\cot^4(x)}\left(8a+2b\cot^4(x)-3b\cot^2(x)+6b\right) + \frac{3\sqrt{a}\sqrt{b}\sqrt{a+b\cot^4(x)}\,\sinh^{-1}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a}}\right)}{\sqrt{\frac{b\cot^4(x)}{a}+1}} + 6(a+b)^{3\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + b*Cot[x]^4)^(3/2),x]
```

```
[Out] (6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + 6*(a
+ b)^(3/2)*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])] - S
qrt[a + b*Cot[x]^4]*(8*a + 6*b - 3*b*Cot[x]^2 + 2*b*Cot[x]^4) + (3*Sqrt[a]*
Sqrt[b]*ArcSinh[(Sqrt[b]*Cot[x]^2)/Sqrt[a]]*Sqrt[a + b*Cot[x]^4])/Sqrt[1 +
(b*Cot[x]^4)/a])/12
```

fricas [B] time = 0.85, size = 1486, normalized size = 11.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(6*((a + b)*cos(2*x)^2 - 2*(a + b)*cos(2*x) + a + b)*sqrt(a + b)*log(
1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2
*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a -
 b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)
) + 3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2*b)*cos(2*x) + 3*a + 2*b)*sqrt(b)
*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2*x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(
2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a
- 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*((8*a + 11*b)
*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a + 5*b)*sqrt(((a + b)*cos(2*x)^2 -
2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)))/(cos(2*x)^2 - 2
*cos(2*x) + 1), 1/12*(3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2*b)*cos(2*x) +
3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*co
s(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))*(cos(2*x) - 1)/(b*cos(2*x) +
 b)) + 3*((a + b)*cos(2*x)^2 - 2*(a + b)*cos(2*x) + a + b)*sqrt(a + b)*log(
1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2
*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a -
 b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)
) - ((8*a + 11*b)*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a + 5*b)*sqrt(((a +
 b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))
)/(cos(2*x)^2 - 2*cos(2*x) + 1), -1/24*(12*((a + b)*cos(2*x)^2 - 2*(a + b)*
cos(2*x) + a + b)*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) +
a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/
(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*
b + b^2 - 2*(a^2 - b^2)*cos(2*x))) - 3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2
*b)*cos(2*x) + 3*a + 2*b)*sqrt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2*x)^
2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(
2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2 - 2
*cos(2*x) + 1)) + 2*((8*a + 11*b)*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a +
 5*b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 -
2*cos(2*x) + 1)))/(cos(2*x)^2 - 2*cos(2*x) + 1), -1/12*(6*((a + b)*cos(2*x)
^2 - 2*(a + b)*cos(2*x) + a + b)*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 -
2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos
(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)
^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x))) - 3*((3*a + 2*b)*cos(2*x)
^2 - 2*(3*a + 2*b)*cos(2*x) + 3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a
+ b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)
)*(cos(2*x) - 1)/(b*cos(2*x) + b)) + ((8*a + 11*b)*cos(2*x)^2 - 8*(2*a + b)
*cos(2*x) + 8*a + 5*b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a +
b)/(cos(2*x)^2 - 2*cos(2*x) + 1)))/(cos(2*x)^2 - 2*cos(2*x) + 1)]
```

giac [B] time = 1.80, size = 445, normalized size = 3.53

$$-\frac{\left(3\,ab+2\,b^{2}\right)\arctan\left(-\frac{\sqrt{a+b}\,\sin(x)^{2}-\sqrt{a\sin(x)^{4}+b\sin(x)^{4}-2\,b\sin(x)^{2}+b}}{\sqrt{-b}}\right)}{2\,\sqrt{-b}}-\frac{\left(a^{2}+2\,ab+b^{2}\right)\log\left(\left|-\left(\sqrt{a+b}\,\sin(x)^{2}-\sqrt{a\,\mathrm{s}}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b + 2*b^2)*arctan(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*si
n(x)^4 - 2*b*sin(x)^2 + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*lo
g(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2
+ b))*(a + b) + sqrt(a + b)*b))/sqrt(a + b) - 1/6*(3*(sqrt(a + b)*sin(x)^2
- sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^5*(5*a*b + 6*b^2) + 8*(
sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^3*
b^3 - 12*(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^
2 + b))^4*(a*b + 3*b^2)*sqrt(a + b) + 12*(a*b^2 + b^3)*(sqrt(a + b)*sin(x)^
2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^2*sqrt(a + b) + 3*(3*
a*b^3 + 2*b^4)*(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*s
in(x)^2 + b)) - 8*(a*b^3 + b^4)*sqrt(a + b))/((sqrt(a + b)*sin(x)^2 - sqrt(
a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^2 - b)^3
```

maple [B] time = 0.29, size = 312, normalized size = 2.48

$$-\frac{b\left(\cot^{4}(x)\right)\sqrt{a+b\left(\cot^{4}(x)\right)}}{6}-\frac{2a\sqrt{a+b\left(\cot^{4}(x)\right)}}{3}+\frac{b\left(\cot^{2}(x)\right)\sqrt{a+b\left(\cot^{4}(x)\right)}}{4}+\frac{3a\sqrt{b}\,\ln\left(\sqrt{b}\,\left(\cot^{2}(x)\right)+\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^4)^(3/2),x)
```

```
[Out] -1/6*b*cot(x)^4*(a+b*cot(x)^4)^(1/2)-2/3*a*(a+b*cot(x)^4)^(1/2)+1/4*b*cot(x
)^2*(a+b*cot(x)^4)^(1/2)+3/4*a*b^(1/2)*ln(b^(1/2)*cot(x)^2+(a+b*cot(x)^4)^(
1/2))-1/2*b*(a+b*cot(x)^4)^(1/2)+1/2*b^(3/2)*ln(b^(1/2)*cot(x)^2+(a+b*cot(x
)^4)^(1/2))+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+cot(x)^2)*b+2*(a+b)^(1/2)*((1+
cot(x)^2)^2*b-2*(1+cot(x)^2)*b+a+b)^(1/2))/(1+cot(x)^2))*a^2+1/(a+b)^(1/2)*
ln((2*a+2*b-2*(1+cot(x)^2)*b+2*(a+b)^(1/2)*((1+cot(x)^2)^2*b-2*(1+cot(x)^2)
*b+a+b)^(1/2))/(1+cot(x)^2))*a*b+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+cot(x)^2)
*b+2*(a+b)^(1/2)*((1+cot(x)^2)^2*b-2*(1+cot(x)^2)*b+a+b)^(1/2))/(1+cot(x)^2
))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cot(x)^{4}+a\right)^{\frac{3}{2}}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^4 + a)^(3/2)*cot(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot(x)\,{\left(b\,{\cot(x)}^4+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)*(a + b*cot(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^4\left(x\right)\right)^{\frac{3}{2}}\cot\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**4)**(3/2)*cot(x), x)
```

3.62 $\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] 1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3670, 1248, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Cot[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*Sqrt[a + b]
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx = -\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Cot[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*Sqrt[a + b]
)
```

fricas [B] time = 0.65, size = 264, normalized size = 6.44

$$\left[\frac{\log\left(\frac{1}{2}\left(a^2+2\,ab+b^2\right)\cos\left(2\,x\right)^2+\frac{1}{2}\,a^2+\frac{1}{2}\,b^2+\frac{1}{2}\left((a+b)\cos\left(2\,x\right)^2-2\,a\cos\left(2\,x\right)+a-b\right)\sqrt{a+b}\sqrt{\frac{(a+b)\cos}{\text{co}}}\right.}{4\sqrt{a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a +
 b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2
 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)
*cos(2*x))/sqrt(a + b), -1/2*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*
cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x
) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 +
 a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x)))/(a + b)]
```

giac [A] time = 0.38, size = 58, normalized size = 1.41

$$\frac{\log\left(\left|-\left(\sqrt{a+b}\,\cos(x)^2-\sqrt{a\cos(x)^4+b\cos(x)^4-2\,a\cos(x)^2+a}\right)(a+b)+\sqrt{a+b}\,a\right|\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="giac")
```

[Out] 1/2*log(abs(-(sqrt(a + b)*cos(x)^2 - sqrt(a*cos(x)^4 + b*cos(x)^4 - 2*a*cos(x)^2 + a))*(a + b) + sqrt(a + b)*a))/sqrt(a + b)

maple [A] time = 0.31, size = 65, normalized size = 1.59

$$\frac{\ln\left(\frac{2a+2b-2\left(1+\cot^{2}(x)\right)b+2\sqrt{a+b}\,\sqrt{\left(1+\cot^{2}(x)\right)^{2}b-2\left(1+\cot^{2}(x)\right)b+a+b}}{1+\cot^{2}(x)}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(a+b*cot(x)^4)^(1/2),x)

[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+cot(x)^2)*b+2*(a+b)^(1/2)*((1+cot(x)^2)^2*b-2*(1+cot(x)^2)*b+a+b)^(1/2))/(1+cot(x)^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{b\cot(x)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="maxima")

[Out] integrate(cot(x)/sqrt(b*cot(x)^4 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)}{\sqrt{b\,{\cot(x)}^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(a + b*cot(x)^4)^(1/2),x)

[Out] int(cot(x)/(a + b*cot(x)^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot{\left(x \right)}}{\sqrt{a+b\cot^{4}{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a+b*cot(x)**4)**(1/2),x)

[Out] Integral(cot(x)/sqrt(a + b*cot(x)**4), x)

3.63 $\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}$$

```
[Out] 1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(3/2)+1/
2*(-a-b*cot(x)^2)/a/(a+b)/(a+b*cot(x)^4)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3670, 1248, 741, 12, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cot[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*(a + b)^(3/
2)) - (a + b*Cot[x]^2)/(2*a*(a + b)*Sqrt[a + b*Cot[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}-\frac{\text{Subst}\left(\int \frac{a}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)}{2a(a+b)}\\
&= -\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}-\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)}{2(a+b)}\\
&= -\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 73, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{(a+b)^{3/2}}-\frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(a + b)^(3/2)
 - (a + b*Cot[x]^2)/(a*(a + b)*Sqrt[a + b*Cot[x]^4]))/2
```

fricas [B] time = 1.36, size = 670, normalized size = 9.05

$$\left[\frac{\left(\left(a^2+ab\right)\cos(2x)^2+a^2+ab-2\left(a^2-ab\right)\cos(2x)\right)\sqrt{a+b}\,\log\left(\frac{1}{2}\left(a^2+2ab+b^2\right)\cos(2x)^2+\frac{1}{2}a^2+\frac{1}{2}b^2+\frac{1}{2}\right.}{4\left(a^4+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cos(2*x)^2 + a^2 + a*b - 2*(a^2 - a*b)*cos(2*x))*sqrt(a
+ b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a +
 b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2
 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)
*cos(2*x)) - 2*((a^2 - b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 + a*b)*
cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^
2 - 2*cos(2*x) + 1)))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*cos(2*x)^2 - 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cos(2*x)
), -1/2*(((a^2 + a*b)*cos(2*x)^2 + a^2 + a*b - 2*(a^2 - a*b)*cos(2*x))*sqrt
(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(-a - b)*sq
rt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*
x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2
)*cos(2*x))) + ((a^2 - b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 + a*b)*
cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^
2 - 2*cos(2*x) + 1)))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*cos(2*x)^2 - 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cos(2*x)
)]
```

giac [A] time = 0.44, size = 111, normalized size = 1.50

$$-\frac{\frac{(a-b)\sin(x)^2}{a^2+ab}+\frac{b}{a^2+ab}}{2\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}}-\frac{\log\left(\left|-\left(\sqrt{a+b}\,\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}\right.\right.\right.}{2(a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*((a - b)*sin(x)^2/(a^2 + a*b) + b/(a^2 + a*b))/sqrt(a*sin(x)^4 + b*sin
(x)^4 - 2*b*sin(x)^2 + b) - 1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin
(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))*sqrt(a + b) + b))/(a + b)^(3/2)
```

maple [B] time = 0.32, size = 248, normalized size = 3.35

$$-\frac{\sqrt{\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}+b\right)a\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)}-\frac{b\ln\left(\frac{2a+2b-2\left(1+\cot^2(x)\right)b+2\sqrt{a+b}\,\sqrt{\left(1+\cot^2(x)\right)^2 b-2\left(1+\cot^2(x)\right)b+a}}{1+\cot^2(x)}\right.}{2\left(\sqrt{-ab}+b\right)\left(\sqrt{-ab}-b\right)\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^4)^(3/2),x)
```

```
[Out] -1/4/((-a*b)^(1/2)+b)/a/(cot(x)^2-(-a*b)^(1/2)/b)*((cot(x)^2-(-a*b)^(1/2)/b
)^2*b+2*(-a*b)^(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/2*b/((-a*b)^(1/2)+b
)/((-a*b)^(1/2)-b)/(a+b)^(1/2)*ln((2*a+2*b-2*(1+cot(x)^2)*b+2*(a+b)^(1/2)*(
(1+cot(x)^2)^2*b-2*(1+cot(x)^2)*b+a+b)^(1/2))/(1+cot(x)^2))+1/4/((-a*b)^(1/
2)-b)/a/(cot(x)^2+(-a*b)^(1/2)/b)*((cot(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(
1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(b\cot(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/(b*cot(x)^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{\left(b\cot(x)^4 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)/(a + b*cot(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a + b\cot^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**4)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a + b*cot(x)**4)**(3/2), x)
```

3.64 $\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}}-\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}}$$

```
[Out] 1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(5/2)+1/
6*(-a-b*cot(x)^2)/a/(a+b)/(a+b*cot(x)^4)^(3/2)+1/6*(-3*a^2-b*(5*a+2*b)*cot(
x)^2)/a^2/(a+b)^2/(a+b*cot(x)^4)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 741, 823, 12, 725, 206}

$$-\frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}}-\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cot[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*(a + b)^(5/
2)) - (a + b*Cot[x]^2)/(6*a*(a + b)*(a + b*Cot[x]^4)^(3/2)) - (3*a^2 + b*(5
*a + 2*b)*Cot[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Cot[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot^2(x)\right)}{6a(a+b)}$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}} - \frac{\text{Subst}\left(\int \frac{3a^2b}{(1+x)\sqrt{a+bx^2}}\,dx, \ldots\right.}{6a^2b(a+b)^2}$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, \ldots\right.}{2(a+b)^2}$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \ldots\right.}{2(a+b)^2}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}}$$

Mathematica [A] time = 0.75, size = 114, normalized size = 0.97

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{3a^2b\cot^4(x)+a^2(4a+b)+b^2(5a+2b)\cot^6(x)+3ab(2a+b)\cot^2(x)}{6a^2(a+b)^2\left(a+b\cot^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*(a + b)^(5/
2)) - (a^2*(4*a + b) + 3*a*b*(2*a + b)*Cot[x]^2 + 3*a^2*b*Cot[x]^4 + b^2*(5
*a + 2*b)*Cot[x]^6)/(6*a^2*(a + b)^2*(a + b*Cot[x]^4)^(3/2))
```

fricas [B] time = 0.74, size = 1365, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(2*x)^4 + a^4 + 2*a^3*b + a^2*b^2 -
4*(a^4 - a^2*b^2)*cos(2*x)^3 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cos(2*x)^2 -
 4*(a^4 - a^2*b^2)*cos(2*x))*sqrt(a + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*
x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*
sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x
)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)) - 4*((2*a^4 + a^3*b - 5*a^2*
b^2 - 5*a*b^3 - b^4)*cos(2*x)^4 + 2*a^4 + 7*a^3*b + 9*a^2*b^2 + 5*a*b^3 + b
^4 - 2*(4*a^4 + 2*a^3*b - a^2*b^2 + 2*a*b^3 + b^4)*cos(2*x)^3 + 12*(a^4 + a
^3*b)*cos(2*x)^2 - 2*(4*a^4 + 8*a^3*b + 3*a^2*b^2 - 2*a*b^3 - b^4)*cos(2*x)
)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*co
s(2*x) + 1)))/(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^
5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(2*x
)^4 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cos(2
*x)^3 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)
*cos(2*x)^2 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^
5)*cos(2*x)), -1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(2*x)^4 + a^4 + 2*a^3*b
 + a^2*b^2 - 4*(a^4 - a^2*b^2)*cos(2*x)^3 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)
*cos(2*x)^2 - 4*(a^4 - a^2*b^2)*cos(2*x))*sqrt(-a - b)*arctan(((a + b)*cos(
2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a
 - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)
*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x))) + 2*((2*a^4 + a^
3*b - 5*a^2*b^2 - 5*a*b^3 - b^4)*cos(2*x)^4 + 2*a^4 + 7*a^3*b + 9*a^2*b^2 +
 5*a*b^3 + b^4 - 2*(4*a^4 + 2*a^3*b - a^2*b^2 + 2*a*b^3 + b^4)*cos(2*x)^3 +
 12*(a^4 + a^3*b)*cos(2*x)^2 - 2*(4*a^4 + 8*a^3*b + 3*a^2*b^2 - 2*a*b^3 - b
^4)*cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2
*x)^2 - 2*cos(2*x) + 1)))/(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*
b^4 + a^2*b^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*
b^5)*cos(2*x)^4 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^
2*b^5)*cos(2*x)^3 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4
+ 3*a^2*b^5)*cos(2*x)^2 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*
b^4 - a^2*b^5)*cos(2*x))]
```

giac [B] time = 0.48, size = 276, normalized size = 2.36

$$-\frac{\left(2\left(\frac{\left(2\,a^3b-a^2b^2-4\,ab^3-b^4\right)\sin(x)^2}{a^4b+2\,a^3b^2+a^2b^3}+\frac{3\left(3\,ab^3+b^4\right)}{a^4b+2\,a^3b^2+a^2b^3}\right)\sin(x)^2+\frac{3\left(a^2b^2-5\,ab^3-2\,b^4\right)}{a^4b+2\,a^3b^2+a^2b^3}\right)\sin(x)^2+\frac{5\,ab^3+2\,b^4}{a^4b+2\,a^3b^2+a^2b^3}}{6\left(a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b\right)^{\frac{3}{2}}}-\frac{\log\left(\left|-\left(\sqrt{a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*((2*a^3*b - a^2*b^2 - 4*a*b^3 - b^4)*sin(x)^2/(a^4*b + 2*a^3*b^2 +
 a^2*b^3) + 3*(3*a*b^3 + b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))*sin(x)^2 + 3*(
a^2*b^2 - 5*a*b^3 - 2*b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))*sin(x)^2 + (5*a*b
^3 + 2*b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))/(a*sin(x)^4 + b*sin(x)^4 - 2*b*s
in(x)^2 + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 +
 b*sin(x)^4 - 2*b*sin(x)^2 + b))*sqrt(a + b) + b))/((a^2 + 2*a*b + b^2)*sqr
t(a + b))
```

maple [B] time = 0.33, size = 602, normalized size = 5.15

$$-\frac{\sqrt{\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)}}{24\left(\sqrt{-ab}+b\right)a\sqrt{-ab}\,\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)^2}-\frac{\sqrt{\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)}}{24\left(\sqrt{-ab}+b\right)a^2\left(\cot^2(x)-\frac{\sqrt{-ab}}{b}\right)}-\frac{\sqrt{\left(\mathrm{co}\right.}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^4)^(5/2),x)
```

```
[Out] -1/24/((-a*b)^(1/2)+b)/a/(-a*b)^(1/2)/(cot(x)^2-(-a*b)^(1/2)/b)^2*((cot(x)^
2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/24/
((-a*b)^(1/2)+b)/a^2/(cot(x)^2-(-a*b)^(1/2)/b)*((cot(x)^2-(-a*b)^(1/2)/b)^2
*b+2*(-a*b)^(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/24/((-a*b)^(1/2)-b)/a/
(-a*b)^(1/2)/(cot(x)^2+(-a*b)^(1/2)/b)^2*((cot(x)^2+(-a*b)^(1/2)/b)^2*b-2*(
-a*b)^(1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/24/((-a*b)^(1/2)-b)/a^2/(cot
(x)^2+(-a*b)^(1/2)/b)*((cot(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(cot(x)
^2+(-a*b)^(1/2)/b))^(1/2)-1/8*(2*(-a*b)^(1/2)+b)/((-a*b)^(1/2)+b)^2/a^2/(co
t(x)^2-(-a*b)^(1/2)/b)*((cot(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(cot(x
)^2-(-a*b)^(1/2)/b))^(1/2)+1/2*b^2/((-a*b)^(1/2)+b)^2/((-a*b)^(1/2)-b)^2/(a
+b)^(1/2)*ln((2*a+2*b-2*(1+cot(x)^2)*b+2*(a+b)^(1/2)*((1+cot(x)^2)^2*b-2*(1
+cot(x)^2)*b+a+b)^(1/2))/(1+cot(x)^2))+1/8*(2*(-a*b)^(1/2)-b)/((-a*b)^(1/2)
-b)^2/a^2/(cot(x)^2+(-a*b)^(1/2)/b)*((cot(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)
^(1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(b\cot(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/(b*cot(x)^4 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{\left(b\,{\cot(x)}^4+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^4)^(5/2),x)
```

```
[Out] int(cot(x)/(a + b*cot(x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a + b\cot^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**4)**(5/2),x)
```

```
[Out] Integral(cot(x)/(a + b*cot(x)**4)**(5/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```